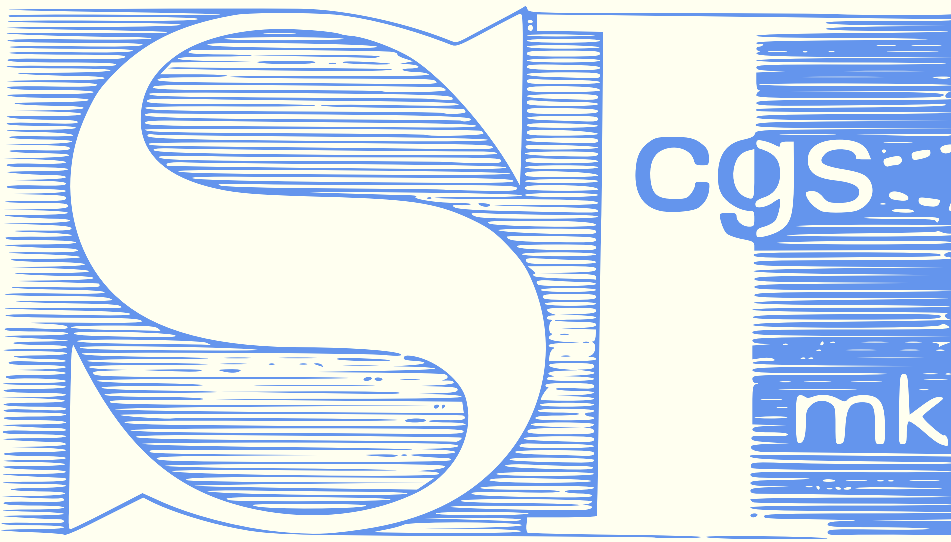
L A SENA

UNITS OF PHYSICAL QUANTITIES AND THEIR DIMENSIONS



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Л. А. СЕНА

ЕДИНИЦЫ ФИЗИЧЕСКИХ ВЕЛИЧИН И ИХ РАЗМЕРНОСТИ

ИЗДАТЕЛЬСТВО «НАУКА»

L. A. SENA

UNITS OF PHYSICAL QUANTITIES AND THEIR DIMENSIONS

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by G. LEIB

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The present book sets out in detail the principles of constructing systems of units, and also the fundamentals of the theory of dimensions. Together with detailed information on the SI system, which is the preferable one at present, a description is given of other systems of units, and also of some non-system units having practical significance.

The book is intended for students of technical colleges and will also be helpful for physics instructors in higher and secondary schools.

На английском языке

TO THE READER

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FOREWORD

More than thirty years ago the author wrote a small book named *Units of Physical Quantities* (in Russian). In 1948 a completely revised edition of this book was published, and further editions with minor corrections appeared in 1949 and 1951.

Not counting the booklet by O. D. Khvolson published in 1887, this was probably the first attempt to systematize the various systems of units and the methods of conversion of units of one system to those of another. The need for such a textbook for students (and this was the purpose of the book) was very great at that time. Students were forced to master the cgs, mk(force)s, metre-ton-second and mks systems, and in addition, when studying electricity and magnetism, to understand the "contradictory" cgse and cgsm systems of dimensions and units. Still more complications were introduced by a great number of various non-system units.

Matters appreciably changed after the introduction of the International System of Units, designated SI, which according to USSR State Standard GOST 9867-61 "has been introduced as preferable in all fields of science, engineering and the national economy, and also in education". In this connection the above book could have been republished, after introducing the essential amendments and additions. The present book, however, basically differs from the original one and has other aims that will be considered below.

Unquestionably, the creation of a system that covers all the fields of measurements and includes a great number of units used in practice should be considered as a progressive step, but in the author's opinion, however, it is not necessary to run to extremes. While the existence of a great diversity of systems of units leads to considerable inconveniences, an attempt to confine all measurements to the

Procrustean bed of a single system will hardly satisfy everyone. Naturally the main, if not the single, requirement for any system of units is its convenience.

The author completely agrees with the opinion of Academician M. A. Leontovich (*Vestnik Akademii Nauk SSSR*, No. 6, 1964) that the cgs system should be retained, and not as one whose "use is allowed", but as one enjoying absolutely all rights.

Other units, in particular some non-system ones, should be retained for a number of purposes, and again for the single reason of their practical convenience in the given field.

If the question were only of introducing the SI system and retaining or not retaining the other systems (in particular, the cgs system), then the publication of this book would hardly be justifiable, and a small article in a journal would suffice, moreover since at present many reference books are available in which the units of practically all the systems and a great number of non-system units have been gathered. The present volume has a different task reflected in its title, of which special mention should be made. The word "dimensions" included in the title underlines the circumstance that together with problems of the systems of units, much attention is given to systems of dimensions. From the very beginning it must be underlined that the word "dimensions" should be related only to the word "units", and not to the word "quantity". In the author's opinion, it is shown quite convincingly in the following that the concept of a "dimension of a physical quantity" is deprived of any meaning whatsoever and can be used only as an abbreviation of the concept "dimension of a unit of the given physical quantity within the limits of the given system of units". Although for the sake of brevity (and also taking into consideration its widespread usage) this abbreviation is frequently used in the book, its true meaning must always be kept in mind.

For the dimension formulas not to be abstract, brief information is given on their use, in particular in the analysis of dimensions and the method of similarity. The usefulness of this will be obvious if it is noted that these methods are finding greater and greater application, while there

are very few publications on the subject, and they are not always available.

Especially great attention is given to the general principles of constructing systems of units and the ways of converting units from one system to another. In considering how individual units are formed, it was found quite expedient to explain the essence of the physical quantity being measured and on the basis of what measurement the given unit is established, rather than give the formal definition of the unit.

In a book covering all the sections of physics, we have to deal with a very great number of quantities. For this reason the selection of the symbols used from among those contained in various recommendations may perhaps have a somewhat arbitrary nature. This selection has been determined in the main by what symbols are used most frequently in textbooks on physics. Since, in accordance with the existing USSR standards and recommendations, capital and small letters may be substituted for each other, those have been selected that seemed to be more convenient*.

Finally, special mention should be given to the liberty taken by the author in naming and designating the technical unit of mass. Instead of designating it t.u.m. or using the designation kgf·s²/m suggested by the dimension formula, he has used the name "inerta" recommended by prof. M. F. Malikov and the symbol "i". If, as contemplated, the technical system of units—mk(force)s—will with time be completely abolished, the question of how to call the unit of mass in this system will vanish. But as long as the system is in use, it seems to be more convenient to have a special name for the unit of such an important quantity as mass.

Without having the object of creating a reference book on units, the author has nevertheless included a great number of tables that may be useful in practical work.

L. A. Sena

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* In the English edition use has been made wherever possible of the symbols recommended by the International Union of Pure and Applied Physics.—*Translator's note.*

CHAPTER ONE

GENERAL CONCEPTS ON SYSTEMS OF BASIC AND DERIVED UNITS

1.1. Physical Quantities and Their Units

Every day we have to deal with a variety of measurements. The measurement of such quantities as length, area, volume, time and weight is encountered at every step and is known to mankind from time immemorial. Without it commerce, erection of buildings, division of land, etc., would be impossible.

Measurements are of especially great significance in engineering and scientific research. It is owing to measurements that such sciences as mathematics, mechanics and physics were begun to be called exact, since they acquired the possibility of establishing exact quantitative relationships to express the objective laws of nature.

Quite often the results of measurements made in a scientific experiment gave a decisive answer to a question of principle posed by science, allowed a selection to be made between two hypotheses, and sometimes even led to the appearance of a new theory or even a new branch of science. Thus, measurement of the velocity of light in various media facilitated the establishment of the wave theory of light, an attempt to measure the absolute velocity of the Earth's motion led to the appearance of the theory of relativity, measurement of the distribution of energy in the black body spectrum gave birth to the quantum theory.

Not a single branch of engineering, from structural mechanics to complicated chemical enterprises, from radio engineering to nuclear power plants, could exist without a developed system of measurements for determining the dimensions and properties of the product, and for establishing conditions of adequate control of the machines and production processes.

The part played by measurements has especially grown owing to the development of automatic control, since automatic systems and computers must receive as their input data the information on various quantities that determine how the process being controlled is proceeding, such as temperature, gas pressure, or flow rate of a fluid.

The tremendous diversity of phenomena encountered in engineering and scientific research correspondingly expands the range of quantities to be measured. The voltage in electric mains, the viscosity of lubricating oil, the elasticity of steel, the refractive index of glass, the power of an engine, the luminous intensity of a lamp and the length of an electromagnetic wave of a radio transmitter are only a few of the countless numbers of quantities measured in science and engineering.

The methods of measurement are also exceedingly diverse. Simple measuring rules and complicated optical instruments serve for measuring length; magnetoelectric, electromagnetic and thermal instruments measure voltage and current; pressure gauges of various types measure pressure, and so on. Regardless of the method used, however, any measurement of a physical quantity consists in comparing the measured magnitude of this quantity with that taken as a unit. For example, when measuring the length of a table, we compare this length with that of another body taken as a unit of length (for example, a metre rule); when weighing a loaf of bread, we find out how many times its weight is greater than that of another body—a definite unit weight, a "kilogram" or "gram", or what fraction of this unit it is.

By measuring the magnitude of a quantity is meant, consequently, finding the relation between this magnitude and the relevant unit. It is this relation, obviously, that will show us the magnitude of the quantity we are interested in.

Since the concept "greater-smaller" can be applied only to homogeneous quantities, it is obvious that only such quantities can be compared. The height of a building can be compared with the distance between two towns, the force tensioning a spring with the weight of a body (i.e., with the force of gravity), but there is absolutely no sense in trying to find out whether the speed of a train is greater

than the length of a pencil, or whether the volume of a cup is greater than the weight of an inkpot. It is just as senseless, of course, to try to measure speed in units of mass, or area in units of force.

For a measurement to be single-valued, it is essential that the ratio between two homogeneous quantities be independent of the unit used to measure them. The overwhelming majority of physical quantities satisfy this condition, which is customarily called the condition of the *absolute magnitude of a relative value*. This condition can be observed if there exists a possibility, at least in principle, for such a quantitative comparison of two homogeneous quantities as to obtain a number showing the ratio between them.

Sometimes such properties are encountered, however, that cannot be characterized by a quantity complying with the above condition. In these cases certain conventional numerical characteristics are introduced that already cannot be considered as units. With the progress of measuring techniques, the possibility can sometimes appear of replacing such conventional characteristics with genuine units. For example, the velocity of wind was previously determined with the aid of the conditional Beaufort wind scale based on the "force of the wind", which was later replaced by measurement of the velocity of the wind in metres per second. At present a definite range of wind velocities has been made to conform to each number of the Beaufort scale. The conventional quantities also include the hardness of materials that is compared with the aid of various scales between which, by the way, there does not even exist an entirely single-valued relation. Although these conventional numerical characteristics of physical properties are not units of measurement, for the reader's convenience the most widely used of them have nevertheless been included in this book.

The question as to how to determine the unit of a quantity being measured, generally speaking, can be answered only arbitrarily. And indeed, the history of material culture knows a tremendous number of various units, especially for measuring length, area, volume and weight. This diversity of units still exists to a certain extent in our time.

The existence of a great number of diverse units naturally created difficulties in international commercial relations, in the exchange of the results of scientific research, etc. As a result scientists of various countries attempted to establish common units that would be in force in all countries. It can be understood, of course, that their aim was not to establish only a single unit for each quantity. Since both great and small values of quantities being measured are encountered in practice, it was found expedient to have corresponding units of different magnitude, both large and small ones, the condition being observed, however, that the conversion of one unit into another be as simple as possible. Such a system of units is the metric system, created in the era of the French Revolution, a system that, as conceived by its authors, was intended to serve "á tous les temps, á tous les peuples" (in all times, for all peoples).

From the middle of the nineteenth century the use of the metric system began to spread quite rapidly. It was adopted in most countries and served as the basis for establishing the units for measurement of various quantities in physics and related sciences. A feature of the metric or, as it is sometimes termed, the decimal system of weights and measures, is that the ratio between the different units of a quantity is equal to an integral (positive or negative) power of ten.

Notwithstanding the obvious advantages and conveniences of the metric system, other local units are used together with it in a number of countries, while in the USA and some other countries the metric system is not an official one at present and is used, and even then not always, only in scientific work. (In Great Britain the transition from Imperial units to metric ones will most likely take place in 1970/75.)

The circumstance that several units may be employed for measuring a quantity leads to the necessity of being able to convert one unit into another. In other words, it is necessary to be able to determine the number measuring the magnitude of a quantity in one unit if the number measuring it in another unit is known. If a given magnitude A of a quantity is measured using the unit α_1 that results

in its numerical value being equal to a_1, then we can write that

$$\frac{A}{\alpha_1} = a_1$$

If, when measuring the same magnitude A using the unit α_2, we get the value a_2, then, correspondingly,

$$\frac{A}{\alpha_2} = a_2$$

or

$$A = a_1\alpha_1 = a_2\alpha_2$$

From the latter expression we get

$$\frac{a_1}{a_2} = \frac{\alpha_2}{\alpha_1} \tag{1.1}$$

This formula expresses the well known tenet that the numerical value of a physical quantity and the unit used to measure it are in inverse proportion, i.e., the greater the unit used to measure the given magnitude of the quantity, the smaller is the number expressing this magnitude. Thus, if the height of a person measured in centimetres is expressed by the number 175, then the same height measured in decimetres will be expressed by 17.5. Many people forget this when dealing with more complicated and less familiar quantities.

For this circumstance not to be forgotten, one must always remember that the symbols used in formulas represent the numbers expressing the magnitudes of quantities in the units used to measure them, rather than the quantities themselves. For a formula to be generally applicable, the symbol of the unit used to measure a quantity is written next to the number expressing its magnitude. For example, we may write: "the height of that person is 17.5 dm" or "the height of that person is 175 cm". The expressions 17.5 dm and 175 cm are equivalent designations of the same height. For this reason we can write

$$17.5\ \text{dm} = 175\ \text{cm}$$

1.2. Direct and Indirect Measurements

As we have already noted, any measurement consists in comparing the given quantity with another homogeneous quantity taken as a unit. There are many instances, however, when such a comparison cannot be made directly. In most cases what we measure is not the quantity of interest to us, but other quantities that are related to it by certain laws and relationships. Not infrequently, to measure a given quantity, it is first necessary to measure several others whose values are used to compute that of the quantity being sought. Thus, to determine the specific weight of a body, its volume and weight must be measured, to find the speed of a vehicle one has to measure the distance it has covered and the time spent to cover it, etc.

In accordance with the above, all measurements are classified as *direct* and *indirect*. The former are generally considered to include all measurements that give the numerical value of the quantity being measured as a result of one observation or reading (for example, on the scale of a measuring instrument). Actually, however, in the majority of such cases we have not direct, but indirect measurements. Indeed, various measuring instruments (voltmeters, ammeters, thermometers, pressure gauges, etc.) give readings in divisions of their scale, so that what we measure directly are only linear or angular deviations of the pointer that indicate the value of the quantity being measured by means of a number of intermediate relationships connecting the deviation of the pointer to the quantity being measured. For example, in a magnetoelectric ammeter a magnetic field determined by the shape and dimensions of a coil and the current flowing through it (which is to be measured) upon interacting with the magnetic field of a magnet induces a torque; the latter is counteracted by the moment of a spring that depends on its mechanical properties, and the coil turns through a certain angle until it reaches a position in which the torque and the moment are in equilibrium. Thus the measurement of an electrical quantity—the intensity of a current—is reduced through a number of intermediate steps to an angular or linear measurement. If a shunt is used in the instrument, there will be an additional

intermediate step between the current to be measured and the directly measured deviation of the pointer.

The reducing of the measurement of the most diverse quantities to linear and angular measurements is characteristic of an overwhelming majority of measuring instruments. This is not accidental, since vision is the most developed of our organs of sense, and hence a comparison of the magnitudes of a quantity that we directly perceive visually is the most clear and convenient for us. Such quantities, naturally, are dimensional ones, first of all lengths and angles. Where no especially high accuracy is required, and with the exception of very small and very great lengths, linear measurements are generally made by directly comparing the length being measured with a unit length and determining the number of times the unit is contained in the given length. In the same way an angle can be measured by superposing a suitable angular unit.

Lengths and angles are not the only quantities, however, that can be measured directly. An area can be measured by superposing on it a suitably selected unit of area, for example, in the form of a square or a triangle. The volume of a liquid can be found using a vessel whose volume is taken as a unit. Time intervals can be measured by directly counting the number of periods of a cyclic process (for example, the oscillations of a pendulum or the changes of day and night).

Indirect methods also are frequently used, however, for measuring the above quantities, namely, the measurement of areas and volumes is reduced to linear measurements, time is read on the face of a time-piece (again linear or angular measures!), etc. If we consider other quantities, it will be easily seen that for most of them methods of direct measurement are not available at present, and we use either special instruments that convert changes in the given quantity into changes in other quantities (mostly lengths and angles), or a number of intermediate measurements from which the quantity being sought is obtained by calculation.

The fact that practically all measurements can be reduced to linear ones does not at all mean that the quantities being measured lose their qualitative feature and are reduced to

length. Actually this simply means that since all phenomena observed in nature occur in space, each of them can be reflected by a relevant spatial movement (expansion of the mercury column in a thermometer, turning of the coil of an electrical measuring instrument, deviation of a beam of electrons in an oscillograph, etc.).

1.3. Basic and Derived Units

Most of the earlier units were established, as a rule, absolutely independently of one another, only units of length, area and volume being an exception in some instances. Conversely, the main feature of modern units is that relationships are established between units of different quantities which are determined by the laws or definitions relating the quantities being measured to one another. Thus, several so-called *fundamental* or *basic* units are conventionally selected, and all the *derived* units are constructed from them.

Since in indirect measurements the value of the quantity being sought is determined from the values of other quantities related to it, the corresponding relationship can be established between the relevant units. The relationships and laws that determine the conditions of indirect measurement can obviously also serve for establishing relations between basic and derived units.

To show how this is done, let us first consider the question of what meaning should be given to the formulas expressing the relationships between various physical quantities. Any relationship between quantities, whether it is a law of nature or the definition of a new quantity, shows how the given quantity changes with a change in the other ones that it is related to. Let us take as an example the relationship between the areas of geometrical figures and their linear dimensions, established by the theorem "the ratio between the areas of geometrically similar figures is equal to the square of the ratio between their corresponding linear dimensions". This relationship can be written as follows:

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \tag{1.2}$$

If by the symbols A_1, A_2, l_1 and l_2 we understand the relevant *quantities*, then only the ratios A_1/A_2 and l_1/l_2 will have a concrete physical meaning. Expression (1.2) can formally be rewritten as

$$\frac{A_1}{A_2}=\frac{l_1^2}{l_2^2} \tag{1.2a}$$

but it will then lose the meaning contained in the original formula (1.2). Indeed, if the square of the term l_1/l_2, which is the ratio between the lengths l_1 and l_2, is also a definite number, then the square of a length, i.e., the product of two lengths, has no meaning.

Matters will be different if we consider that the symbols in formula (1.2) denote not the quantities themselves, but the numbers by means of which these quantities are expressed when definite units are selected for the corresponding quantities, in our case for length and area. Here each symbol already represents the ratio between the given quantity and another homogeneous quantity taken as a unit. With such a conception of the symbols in the expression of any physical law, they can be multiplied, divided, raised to a power, etc., while the formulas themselves can be subjected to various transformations. In particular, formula (1.2) can be written in a different form, for example

$$\frac{A_1}{l_1^2}=\frac{A_2}{l_2^2} \tag{1.2b}$$

For this reason in mathematical formulation and in the exposition of various physical phenomena and the laws they follow, and in their theoretical analysis, the symbols of physical quantities are understood to denote the numbers by means of which these quantities are expressed in the units used. In the following we also shall always adhere to such a meaning of the symbols.

From this viewpoint formula (1.2b) can be expressed in words as follows: "for geometrically similar figures the ratio between the number expressing the area of a figure and the square of the number expressing the corresponding linear dimension of the figure is a constant quantity". Upon denoting this constant by C, we can write the following equation instead of expression (1.2b):

$$A=Cl^2 \tag{1.3}$$

where the factor C depends, on one hand, on the shape of the geometrical figure being measured, and, on the other, on the units of length and area used*.

As mentioned earlier, these units can in principle be selected absolutely independently of each other. The existence of a relationship between the magnitudes of the area and linear dimensions of the figure, however, makes it possible to relate the units of area to the units of length, i.e., to make the unit of area a derivative of the unit of length. For this purpose it is essential to agree that the unit of area will be the area of a certain figure whose linear dimension is equal to a definite conventionally accepted number of units of length. In geometry this is usually done as follows. After taking any unit of length, for example, a metre, as the basic one, the unit of area is taken equal to the area of a square whose side is equal to the selected unit of length, in our case one metre. This unit of area, as is well known, is called a "square metre" (sq m). Assuming in formula (1.3) that $l = 1$ m, we can write

$$1 \text{ sq m} = C(1 \text{ m})^2 \tag{1.3a}$$

whence

$$C = 1 \text{ sq m/m}^2$$

and formula (1.3) can accordingly be written as

$$A \text{ sq m} = \left(1 \frac{\text{sq m}}{\text{m}^2}\right)(l \text{ m})^2 \tag{1.3b}$$

If, without changing the units of length and area, we rewrite formula (1.3) for a circle, we get

$$A \text{ sq m} = \left(\frac{\pi}{4} \frac{\text{sq m}}{\text{m}^2}\right)(l \text{ m})^2 \tag{1.3c}$$

(where l is the diameter of the circle), since here the factor C will be equal to $\frac{\pi}{4} \frac{\text{sq m}}{\text{m}^2}$.

* Here and below we shall use the symbol C for the general designation of a factor of proportionality in the formulas of physical laws and definitions regardless of its concrete value. In separate instances, when this will be found expedient, the factor C will be provided with a subscript.

It is obvious that the accepted relationship between a unit of area and one of length can be retained with any other unit of length. Here formula (1.3) can be rewritten for a square as

$$A = l^2 \tag{1.4}$$

and for a circle as

$$A = \frac{\pi}{4} l^2 \tag{1.4a}$$

Formulas (1.4) and (1.4a) can be expressed in words as follows: "if the area of a square whose side is equal to a unit of length is taken as the unit of area, then the number expressing the area of any square will be equal to the second power of the number expressing the length of its side, while the number expressing the area of any circle will be equal to the product of $\pi/4$ and the second power of the number expressing its diameter".

Of course such formulations are exceedingly cumbersome and for this reason they are replaced by shorter ones, namely, "the area of a square is equal to the second power of its side", and "the area of a circle is equal to the second power of its diameter multiplied by $\pi/4$", taking it as granted that we are speaking of numbers used to express the relevant quantities with their units appropriately selected.

The example considered above clearly shows the method used to establish a derived unit. To do this it is necessary to:

1. Select the quantities whose units are to be taken as the basic ones.
2. Establish the dimensions of the basic units.
3. Select the defining relationship between the quantities measured by the basic units and the quantity for which a derived unit is to be established.
4. Equate to unity (or any other constant number) the factor of proportionality in the defining relationship.

It is obvious that the symbols of all the quantities in the defining relationship should designate not the quantities themselves, but their numerical values.

In the following, the quantities measured by basic units will be called *basic* ones, and those measured by derived units *derived* ones. It must be stressed here that these generally used terms—"basic quantities" and "derived quanti-

ties"—should by no means be understood in the sense that the former have some privileges or advantages of principle over the latter. A quantity that in one selection has been taken as a basic one may become a derived one in another selection, and vice versa.

Defining relationships used to establish derived units should be written, for convenience, in the form of an explicit functional dependence of the derived quantity on the basic ones.

Derived units established as described above can be further used for developing new derived units. This is why the defining relationships, in addition to basic quantities, may include derived ones whose units were established previously.

Let us explain the above with examples. The unit of velocity can be established by using the relationship between distance and time, written as

$$v = C\frac{dl}{dt} \tag{1.5}$$

For the particular case of uniform motion expression (1.5) can be replaced by

$$v = C\frac{l}{t} \tag{1.5a}$$

where, as previously, C is a factor depending on the units of distance, time and velocity selected. As in the example with the establishment of the unit of area, the unit of velocity may be selected regardless of the units of distance and time. In practice, however, the unit of velocity is determined as a derivative of these units, which are taken as the basic ones. The factor C is assumed to be equal to unity, so that the unit of velocity is determined as the velocity of such uniform motion when a distance equal to a unit of length is covered in a unit of time. In a similar way the unit of acceleration can be established with the aid of the formula defining it, namely

$$a = C\frac{dv}{dt} \tag{1.6}$$

which for uniformly accelerated motion takes the form

$$a = C\frac{v_2 - v_1}{t} \tag{1.6a}$$

Here the difference $v_2 - v_1$ denotes the change in velocity during the time t. Assuming, as previously, that $C = 1$, we obtain the derived unit of acceleration, defined as the acceleration of such uniformly accelerated motion in which the velocity grows by one unit in a unit of time. In this definition together with a basic unit (time) use is made of a previously established derived unit (for velocity).

Let us consider another example—the establishing of the unit of force. Like the units of any other quantities, the unit of force can be established independently of others and even be taken as a basic unit. Most frequently, however, the unit of force is determined as a derived one on the basis of Newton's second law. Writing this law as

$$f = Cma \tag{1.7}$$

(where m is the mass of a material point) and taking C equal to unity, we obtain the definition of a unit of force as such a force that imparts to a material point with a mass equal to a unit of mass (taken as a basic unit) an acceleration equal to a unit of acceleration (previously determined as a derived unit).

If, for example, we take the metre (m) as the unit of length, the second (s) as the unit of time, and the kilogram (kg) as the unit of mass, then a metre per second and a metre per second per second will respectively be the derived units of velocity and acceleration. The force imparting an acceleration of one metre per second per second to a mass of one kilogram is taken as the unit of force, called a *newton* (N). Here, obviously, the factor C will have the value

$$C = 1\,\frac{\mathrm{N\cdot s\cdot s}}{\mathrm{kg\cdot m}}$$

and Newton's second law can be written in the following form:

$$f_N = \left(1\,\frac{\mathrm{N\cdot s\cdot s}}{\mathrm{kg\cdot m}}\right)(m\ \mathrm{kg})\left(a\,\frac{\mathrm{m}}{\mathrm{s\cdot s}}\right) \tag{1.7a}$$

If we take the centimetre (cm) as the unit of length and the gram (g) as the unit of mass, retaining the second as the unit of time, then the corresponding unit of force—the dyne (dyn)—is determined as the force imparting an acceleration of one centimetre per second per second to a mass of one gram. Here the factor of proportionality is

$$C = 1\,\frac{\mathrm{dyn\cdot s\cdot s}}{\mathrm{g\cdot cm}}$$

and Newton's second law can be written as follows:

$$f_{\mathrm{dyn}} = \left(1\,\frac{\mathrm{dyn\cdot s\cdot s}}{\mathrm{g\cdot cm}}\right)(m\ \mathrm{g})\left(a\,\frac{\mathrm{cm}}{\mathrm{s\cdot s}}\right) \qquad (1.7\mathrm{b})$$

In writing the formulas of physical relationships, such a designation of the factor C, containing in essence the definition of the derived unit, is usually omitted, so that, for instance, Newton's second law becomes

$$f = ma \qquad (1.7\mathrm{c})$$

It should be remembered, however, that actually the factor of proportionality is "invisibly" present in every such formula. Forgetting this often leads to misunderstandings and serious errors.

The method of establishing a derived unit is reflected in the designation of the unit, which is constructed by grouping the units on which its determination is based in accordance with the usual rules of algebra. The units of area m^2 (square metre), acceleration m/s^2 (metre per second per second), etc. are formed in this way.

An exception here are the units that are given their own names, such as the dyne and newton. The symbols of these units, the same as those of the basic units, may be included in the compound designation of a derived unit. The unit of pressure N/m^2 (newton per square metre) can be mentioned as an example.

1.4. Constructing a System of Units

A complex of basic and derived units forms a *system of units*. For constructing a system of units it is necessary, obviously, to select several basic units and establish the

derived units of all the other quantities we are interested in with the aid of defining relationships. The latter may be of two types. One includes relationships that are essentially a definition of the new quantity, as, for example, the formula of acceleration (1.6) or that of work

$$dW = Cf \cdot dl \cos(\widehat{f, dl}) \qquad (1.8)$$

The other type includes relationships between the quantities being investigated that have been established experimentally or theoretically. These include the law of universal gravitation and Coulomb's law on the interaction of electrical charges. The division of relationships into "definitions" and "laws" is, however, not absolute and depends on how the given concrete problem is approached. This, nevertheless, does not play any significant part in the determination of new units, since in both instances the relationships are given as formulas connecting the given quantity to others for which units have already been established.

In connection with the outlined program for the formation of derived units and constructing a system of units, there naturally arises a question as to what extent we are free in selecting the basic quantities (in particular, their number), the defining relationships, and the factors of proportionality. The arbitrariness with which the size of the basic units is selected will hardly raise any doubts. The existence of systems in which different units of length (the metre and the centimetre), and also different units of mass (the kilogram and the gram) are used as the basic ones clearly illustrates that in principle this selection is quite arbitrary.

It is easy to show further that there is also complete arbitrariness in selecting the factors of proportionality in the defining relationships. For this purpose let us return to the example of establishing the unit of area considered earlier. After selecting the metre as the unit of length, we established as the unit of area the square metre—the area of a square whose side is equal to one metre. This method of establishing the derived unit of area, although it does have certain practical advantages, is, however, not at all compulsory. We can, for example, take the area of a circle whose diameter is equal to one metre as the unit of

area. Let us call this unit a "round metre" (rd m). This method of establishing the unit of area is equivalent to changing the factor in formula (1.3c) from $\frac{\pi}{4}\frac{\text{sq m}}{\text{m}^2}$ to $\frac{\text{rd m}}{\text{m}^2}$, and in formula (1.3b) from 1 $\frac{\text{sq m}}{\text{m}^2}$ to $\frac{4}{\pi}\frac{\text{rd m}}{\text{m}^2}$. Formulas (1.4) and (1.4a) will correspondingly become

$$A=\frac{4}{\pi}l^2 \text{ (the area of a square)} \tag{1.9}$$

and

$$A=l^2 \text{ (the area of a circle)} \tag{1.9a}$$

It should be noted that the measurement of area in round metres instead of square ones is not unnatural or, moreover, unlawful. Here we can only speak of the practical advantages of the relevant unit. (It should be noted in passing that in the USA round units of area are employed for measuring the cross-sectional area of pipes, tubes, round beams and girders, etc.) If the round metre were used as the unit of area instead of the square metre, then the formulas expressing the areas of various geometrical figures would obviously change. For instance, the area of an equilateral triangle would be expressed by the formula

$$A=\frac{\sqrt{3}}{\pi}l^2 \tag{1.9b}$$

Regardless of how the unit of area has been determined, a square metre or a round metre, its designation will be m². This shows that the symbol of a derived unit including the symbols of basic units cannot in itself give any indication of the magnitude of this derived unit. It is appropriate to note here that the general use of square units of area and correspondingly cubic units of volume has also given rise to the names of the second and third powers of numbers (the "square" and the "cube" of a number).

In the example considered above, the different defining relationships (the area of a square and that of a circle) led only to a change in the numerical coefficients in the formulas, since in essence we used the same geometrical law connecting the areas of similar figures with their linear dimensions.

We shall now show that it is possible to select essentially different defining relationships for establishing the derived unit of a quantity, using as an example the establishing of a unit of force. As mentioned earlier, for this purpose use is generally made of Newton's second law, which mathematically can be written as

$$f = Cma \tag{1.7}$$

The factor of proportionality C that depends on the units selected for the quantities in formula (1.7) will be called the inertial constant and denoted by C_i. In all the systems of units used in practice, the inertial constant is taken equal to unity, owing to which the generally accepted abridged formulation of Newton's second law has become possible, viz., "the force is equal to the product of the mass and the acceleration".

While retaining the units of length, mass and time as the basic ones we, however, are not limited only to Newton's second law for determining the unit of force. We also have at our disposal the law of universal gravitation, according to which any two material points are attracted to each other with a force directly proportional to the masses of these points and inversely proportional to the square of the distance between them. This law can be written as follows:

$$f = C_g \frac{m_1 m_2}{r^2} \tag{1.10}$$

where r is the distance between the points being attracted, and C_g is what is called the gravitational constant whose numerical value depends on the units selected. It should be noted that the generally accepted symbol for the gravitational constant is G. Here we have retained the symbol C to stress that this constant belongs to the category of factors of proportionality in expressions of physical laws. Experience shows that if the kilogram, metre and second are taken as the basic units, and the derived unit of force, the newton, is determined from Newton's second law, then the gravitational constant C_g will be equal to $6.67 \times \times 10^{-11}$ N$\cdot$m^2/kg^2.

With the same basic units of length, mass and time (m, kg and s), however, we can use formula (1.10) as the

defining relationship and, assuming $C_g = 1$, determine the unit of force as the force of mutual attraction of two material points whose masses are equal to unity with the distance between the points equal to the unit of length. It is obvious that if we follow this path we shall have to retain an inertial constant differing from unity in the expression of Newton's second law. It is easy to see that the new "gravitational" unit of force will equal 6.67×10^{-11} N, and the *inertial constant* will be equal to $C_i = 1.5 \times 10^{10}$ grav. force unit$\cdot$s^2/(kg$\cdot$m).

Although the gravitational method of establishing the unit of force is encountered quite rarely (mainly in astronomy), it is no less lawful as a result, however, than the usual "inertial" method. In the following, when considering the units of electrical and magnetic quantities, we shall become acquainted with how the selection of different defining relationships for establishing the unit of the same quantity led in due course to the construction of different, but fully equivalent systems of units.

It is thus evident that there are no strict limitations in the selection of the defining relationships, as in the selection of the values of the basic units and the numerical factors in the defining relationships. What is controversial to the greatest extent and must be considered in greater detail is the question of the number of basic units, on which there exist two diametrally opposite opinions.

According to one of them the number of basic units has been established for us by nature and is determined by the character of the phenomena to be considered. Even "philosophical" considerations that each new quality must be characterized and measured by a new basic unit are given as a substantiation of such an opinion. It is stated that to describe all the phenomena of mechanics it is necessary and sufficient to have three basic units. In investigating other physical phenomena it is necessary, in addition to these three basic units, to introduce for each field of physics at least one additional unit for a quantity characteristic of that field. For example, when dealing with heat such a quantity may be the temperature, with electricity—the electric charge (the quantity of electricity) or the current intensity, and so on.

The supporters of the opposite viewpoint, which the author of this book belongs to, consider that the arguments given above are not substantiated for the following reasons. The qualities of the material world are infinitely varied, and if we consider that each quality is characterized by a quantity whose unit should be a basic one, then the number of such units will also be infinitely great. Indeed, the concept of area cannot be deduced from that of linear length and, consequently, the unit of area should be a basic one. The same relates to the unit of volume. In this case the units of electrical charge, of the induction of a magnetic field, of force, of energy and, naturally, of any other physical quantity should be independent, basic ones. On the other hand, a supposedly "philosophical" substantiation of the fact that only one unit should be the basic one is also possible, since there exists a mutual relationship between all phenomena of nature, reflecting the unity of matter. Thus, attempts to substantiate the number of basic units on the basis of "general philosophical considerations" lead to two diametrally opposite conclusions, namely, the number of basic units should be infinitely great or, on the contrary, there should be only one basic unit.

Both these conclusions are erroneous. The matter is that while the *physical quantities* reflecting the actual properties of the world surrounding us are indeed infinitely varied and cannot be reduced to one another, units of measurement are not by themselves objects of nature, and are only an auxiliary apparatus for studying it. The laws of nature do not in any way change their objective character when one set of units is replaced with another one, in the same way as no mathematical laws change when the decimal system of counting is replaced with the binary one. For this reason the main requirement which a system of units should meet is that it must be convenient as possible for practical calculations.

Assuming that the number of basic units is in principle quite arbitrary and can be both increased and decreased, we do not at all presume that qualitatively different physical phenomena can be reduced to one another, in particular to purely mechanical phenomena. The *measurement*

of different physical quantities, however, can be reduced to the *measurement* of mechanical or even geometrical quantities, and, consequently, there is a possibility of making the corresponding units derived ones.

To show more clearly the arbitrariness of the number of basic units, let us turn again to the example on the establishment of the unit of force considered above. We saw that Newton's second law and the law of universal gravitation can be used with equal right as the defining relationship for this purpose. There is also a third possibility, however, namely, by combining both laws, we can use as the defining relationship the resulting equation expressing the combined law. This equation can be expressed as

$$a = C\frac{m}{r^2} \tag{1.11}$$

The meaning of this equation consists in that the acceleration acquired by a material point under the influence of attraction to another fixed material point with a mass of m and at a distance of r from the first one is proportional to the mass m and inversely proportional to the square of the distance r. The factor of proportionality C in formula (1.11) is in essence the ratio between the gravitational constant and the inertial one. The combination of Newton's second law and the law of universal gravitation into a single law is not at all artificial, as it may seem to be at first sight. It can be easily seen that formula (1.11) is equivalent to Kepler's third law (which is a law of nature discovered by experiment, and, as should be noted in passing, was discovered before Newton's laws). Indeed, let us assume for the sake of simplicity that the planets move along circular orbits with a constant angular velocity ω or a sidereal period (period of revolution) T and substitute for the centripetal acceleration in formula (1.11) its expression

$$a = \omega^2 r \tag{1.12}$$

From the resulting expression, after the relevant transformations, it is simple to see that the squares of the periods of complete revolution of the planets around the Sun are proportional to the cubes of the radii of their orbits. A more general treatment taking into account that the planets

move along ellipses leads to the same expression, the only difference being that the square of the mean distance from a planet to the Sun is substituted for the square of the radius of its orbit. Kepler's law, however, is true not only for the motion of the planets about the Sun, but also for the motion of satellites around their planets. Using the relationship between the angular velocity and the period of revolution $\omega = \frac{2\pi}{T}$, we can, by combining Newton's second law and the law of universal gravitation, write Kepler's law as follows

$$T^2 = 4\pi^2 C R^3 \tag{1.13}$$

where the factor C is the same for all the planets moving around the Sun; it is also the same, but of a different magnitude, for all the satellites of Jupiter; it is the same, but again of a different magnitude, for all the satellites of Saturn; it is the same, but also of a different magnitude, for the Moon and artificial satellites of the Earth, etc. In other words, this factor is the same for any bodies revolving about a single common centre, but is different for different heavenly bodies serving as the centre of rotation. It is easy to see that this factor is inversely proportional to the mass of the body in the centre of the given system (in the above examples the Sun, Jupiter, Saturn, and Earth, respectively). By separating the mass from the factor C, Kepler's law can be rewritten as follows:

$$T^2 = \frac{C'}{m} R^3 \tag{1.14}$$

where C' is now a universal factor that depends only on the units selected.

Formula (1.14) can serve as the defining relationship for establishing the unit of mass as a *derived* unit if we assume that $C' = 1$.

A system of units constructed in this way will have only two, and not three, basic units, i.e., length and time. Of great significance here is the fact that both the inertial and the gravitational constants become dimensionless, and in particular can be made equal to unity. This, of course, is not accidental. If we analyse how we have found it possible

to reduce the number of basic units, it will be easy to see that this was achieved by making both constants dimensionless.

It thus follows that the number of basic units is closely related to the number of factors in the expressions of physical laws and definitions. These factors of proportionality, such as the gravitational and inertial constants, determined depending on the basic units and the defining relationships selected, have been called universal constants. In this they differ from the so-called specific constants characterizing various properties of separate substances (these properties include molecular weight, critical temperature, and electric permittivity).

In principle, universal constants are always present in the expressions of all physical laws and definitions, but by appropriately selecting the units we can equate a certain number of them to unity (or any other constant number). Consequently, the greater the number of basic units used for constructing a system, the greater the number of universal constants that will be present in the formulas. A reduction in the number of basic units is always accompanied by a reduction in the number of universal constants. It is natural to ask whether a further reduction in the number of basic units to one (or even to none!) is possible in this way.

Below, in dealing with the methods of constructing a system of units of electrical and magnetic quantities, we shall show that it is not difficult to reduce the number of basic units to one. And what is more, one of the equations of atomic physics, determining the so-called fine-structure constant, makes it possible to construct a system without any basic units whatsoever. At first sight this seems to be a paradox. As we shall see in the following (Sec. 9.8), however, there actually exists such a possibility. It has been found that if we equate certain constants to unity, we shall thus rigidly fix the dimensions of the units of all physical quantities.

Having analysed the fundamental principles of constructing a system of units, we have become convinced that there is almost unlimited arbitrariness in selecting the ways of doing this.

This arbitrariness is such only theoretically, however. Since a system of units is a sort of apparatus intended for facilitating calculations in science and engineering, it must comply with a number of practical requirements. From this viewpoint the method of constructing a system of units and, in particular, the number of basic units are not a matter of indifference and to a certain extent are limited.

A too great number of basic units inevitably results in a great number of universal constants in the physical formulas, which makes it difficult to remember them and leads to more complicated calculations. In addition, enormous work would be required to establish standard specimens of all the basic units. The accuracy which these standards would be established with would be different, and as a result the universal constants in the formulas expressing physical laws and definitions would also be of different accuracy. On the other hand, a too small number of basic units would limit the possibilities of constructing the derived units to such an extent that a considerable part of the latter would inevitably be either too great, or too small, and, consequently, inconvenient for practical work. It should be noted in passing, however, that the expression "dimension of a unit convenient for practical work" has at present become somewhat diffused owing to the fact that the range of the dimensions of quantities encountered in science and engineering is exceedingly broad. For instance, in nuclear physics lengths of the order of 10^{-15} m are encountered, and in astronomy of the order of 10^{22}-10^{26} m. The capacities of electric power plants exceed 10^{9} W, while the power of a signal that can be picked up by a radar station is less than 10^{-16} W. In scientific research the values of pressure range from below 10^{-15} to tens and hundreds of thousands of atmospheres.

There are also other practical considerations that make a system with a too small number of units quite unsuitable. Some of them will be considered below when setting out the fundamental conceptions of the *analysis of dimensions*.

With a view to what has been said above, it has been found good practice to construct a system having about three to six basic units.

1.5. Selection of Basic Units

It is expedient to select such quantities as the basic ones that reflect the most general properties of matter. Since space and time are forms of existence of matter, it is natural to include length and time in the basic units. Seeing that mass is one of the most general characteristics of matter, a unit of mass is taken as a third basic unit in most systems. Such a system constructed on these three units (length, mass, and time) was first proposed by Gauss and named the absolute system by him.

Later this concept gradually lost its non-ambiguity. By absolute were sometimes meant systems constructed on quite definite units of length, mass and time (centimetre, gram and second), sometimes, on the contrary, this name was given a broader meaning, considering as absolute any system having a certain limited number of basic units and including all the remaining units from the field of geometry, mechanics, electricity and electromagnetism as derived ones. At present the term "absolute system" is used less and less frequently, moreover since, as we have seen, there is no criterion that would make it possible on the basis of considerations of principle to give preference to any definite system and attach such a committing name to it.

Among the existing systems of units the most favoured ones are those based on units of the three quantities indicated above, some of the systems limiting the number of basic units to only these three. The so-called technical system covering only geometrical and mechanical measurements also has three basic units, but here the third unit is that of force, and not mass. By decisions of the 10th and 11th General Conferences on Weights and Measures there has been introduced the International System of Units, designated SI, that covers the measurement of all mechanical, electrical, heat and light quantities. The basic units of this system are length, time, mass, intensity of electric current, temperature and luminous intensity. In accordance with USSR State Standard GOST 9867-61, the International System of Units has been introduced as preferable in all fields of science, engineering and the national economy, and also in educational institutions. Together with this

system, however, the use of certain other systems, and also of a number of non-system units, is also allowed.

Of great importance in establishing the basic units is the possibility of retaining the constancy of a unit, of checking and reproducing it, and if lost, of restoring it. For this reason the trend appeared of relating the basic units to quantities encountered in nature.

In the era of the Great French Revolution a special committee including the most renowned French scientists of that time (Borda, Condorcet, Laplace and Monge), and created in May 1790 by edict of the National Assembly, proposed to take as the unit of length one ten-millionth of a quarter of the Earth's meridian. On March 30, 1791, the committee's proposal was approved, and it began to determine the accepted unit. As a result of its work, in 1799 there was introduced in France the "Mètre vrai et difinitif" ("the genuine and final metre") that served as the basis of the metric system. The prototype of the metre was a specially made platinum bar, which at present is kept in the French National Archive (the "archive metre").

Together with the metre a unit of weight, the kilogram, was introduced that was originally defined as the weight of a cubic decimetre of water at 4°C. In the same way as a standard bar was made for retaining the metre, a standard weight, the prototype of the kilogram, was made for retaining the kilogram.

The second, determined as 1/86 400 of the mean solar day, was taken as the unit of time.

The increase in the accuracy of measurements as a result of the progress of measuring instruments and techniques made it possible, however, to find that there was a slight, but quite measurable discrepancy between the selected units and the prototypes made for them. The only exception was the second, which owing to the high accuracy of astronomical measurements remained practically unchanged and required only greater accuracy of the formulation.

In this connection the problem arose of whether to make new prototypes or to accept the existing discrepancies and take as the legal units the measures determined by the existing prototypes.

Besides the fact that changing of the latter would in itself involve enormous difficulties and inconveniences, there would be no guarantees that a new more accurate determination would not require them to be changed again. For this reason it was found necessary to accept the prototypes as the basic legal standard units.

Thus the following units were established, which were taken as the basic ones:

the unit of length—the metre (m)—defined as the distance between the centres of marks made on a bar of a platino-iridium alloy at 0°C. The platino-iridium alloy was selected because it has a very low coefficient of thermal expansion, while the shape of the cross section of the bar (Fig. 1) met the requirement of the smallest possible deflection,

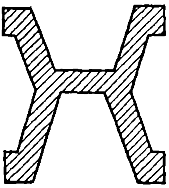

Fig. 1

the unit of mass—the kilogram (or kilogram-mass) (kg)— the mass of a platino-iridium weight,

the unit of force—the kilogram-force (kgf)*—the weight of the same prototype at its place of storage in the International Chamber of Weights and Measures at Sèvre (near Paris);

the unit of time—the second, determined, as previously, as 1/86 400 of the mean solar day. In astronomy and related fields the stellar second was taken defined as 1/86 400 of a stellar day. Since owing to the rotation of the Earth around the Sun the number of stellar days in a year is greater by one than the number of solar days, then the stellar second is 0.99726957 of the solar second.

The following systems were constructed on the basis of the above units and their decimal fractions:

the cgs system, whose basic units are the unit of length—the centimetre (cm), equal to one-hundredth of a metre; the unit of mass—the gram or gramme (g), equal to one-thousandth of a kilogram; and the unit of time—the second (s).

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* A number of other names were proposed for this unit, such as kilograv and kilopond, but none of them were introduced to any considerable extent. The name kilopond is used in Germany.

The mk(force)s system, whose basic units are the unit of length—the metre (m); the unit of force—the kilogram-force (kgf); and the unit of time—the second (s).

The mk(force)s system included only geometrical and mechanical units, while the cgs system also covered electrical and magnetic measurements. The latter system branched out into two independent ones, one of which was based on electrostatic, and the other on electromagnetic interactions. Correspondingly, the first of them was named the electrostatic system (cgse), and the second the electromagnetic system (cgsm). The units of both these systems, however, were found to be inconvenient for practical work, and as a result auxiliary practical units were introduced for measuring quantities related to the process of flow of a current (current intensity, potential difference and electromotive force, resistance, work, power, etc.).

Originally these units did not form a harmonious system, and could not even be used for solving problems involving electrostatic and electromagnetic phenomena. In 1902 the Italian engineer Giorgi proposed to so extend the system of practical units as to make it just as universal as the cgs system, i.e., covering all measurements in the fields of mechanics, electricity and electromagnetism. Since this system was to include practical units already being extensively used, it was found possible to retain the latter only on condition of introducing at least one additional universal constant, which is equivalent to including one of these units in the number of basic ones. It was intended to use in this capacity the unit of one of the following quantities: the amount of electricity (electric charge), current intensity, potential difference, resistance, capacitance, inductance, magnetic flux, and magnetic permeability. A special name "magn" was even proposed for the latter unit.

After a number of discussions, with a view to the metrological considerations of the convenience and reliability of reproducing the unit, the decision was reached to take the unit of current intensity, the ampere, as the fourth basic unit. The definition of the ampere appreciably differs in its nature from the definition of the other basic units—the metre, kilogram and second. The matter is that the ampere was first introduced as one-tenth of the derived unit

of current intensity in the cgsm system. For this reason, although the ampere has been promoted to the rank of basic units, it is in essence defined as a derived unit. According to the accepted definition, "the ampere is the intensity of an unchanging current which, upon flowing along two parallel conductors of infinite length and with a negligibly small round section, arranged at a distance of one metre from each other in a vacuum, would induce between these conductors a force equal to 2×10^{-7} newtons per metre of length". We shall consider this definition in greater detail and explain it in the chapter on the units of electrical and magnetic quantities.

In connection with the fact that the metre and kilogram were defined not as natural values, but according to prototypes, one of the advantages of the metric system, namely, its intactness and the possibility of exact reproduction of the units, has been lost. A further increase in the accuracy of measurements made it possible to partly return to the establishment of the basic units in accordance with the measurement of natural quantities. The determination of the unit of mass, the kilogram, was retained according to the international prototype, while it was found possible and most expedient to relate the length of the metre to the wavelength of a definite line of the spectrum, for which purpose the orange line of krypton was taken. Since natural krypton contains six isotopes, whose spectral lines, although very slightly, differ from one another, the definition of the metre through wavelength is given more precisely by indicating that the isotope of krypton with a mass number of 86 ($_{36}Kr^{86}$) is taken as the source. The spectral line taken corresponds to the transition of an electron in an atom of krypton between the quantum states that in spectroscopy are designated by the symbols $2p_{10}$ and $5d_5$. According to definition, a metre contains 1 650 763.73 wavelengths of this spectral line in a vacuum.

The definition of the second has also been defined somewhat more accurately, since the improvement in the accuracy of measuring time made it possible to establish a certain lack of constancy in the mean day. The new definition of the second is based on the so-called tropical year—the interval of time between two vernal equinoxes. According

to the new definition, a second is 1/31 556 925.9747th part of the tropical year beginning at 12 noon on December 31, 1899. (According to the astronomical recording of time we should write "12 noon on January 0, 1900".) The indication of a definite year has the aim of taking account of the circumstance that the tropical year itself decreases by about 0.5 second in a century.

The progress of molecular and atomic radiation spectroscopy has made it possible to establish a sufficiently accurate relationship between the unit of time and the period of oscillations corresponding to a definite spectral line. For this reason a decision of the 13th General Conference on Weights and Measures (1967) gave a new definition of the second, according to which it is the duration of 9 192 631 770 periods of radiation corresponding to the transition between two hyperfine levels of the principal state of an atom of $_{55}Cs^{133}$ (the isotope of cesium with a mass number of 133).

It should be noted that in principle a unit of mass could also be determined not by the mass of a prototype, but by relating it to the mass of an atomic particle (for example, a neutron). Unfortunately, the accuracy of determining atomic masses is at present inferior to that of measuring a mass by weighing.

With respect to the cgs system, then, since it is constructed on three basic units (length, mass, and time), it completely covers all geometrical, mechanical, electrical, and magnetic measurements. It has been found most convenient to use such a variant of the system in which electrostatic quantities are measured in cgse units, and magnetic ones in cgsm units. This system has been named the symmetrical, or Gaussian system of units and designated cgs. In principle this system could be applied, naturally, for any other, in particular thermal and light, measurements, for which purpose the relevant quantities should be related by defining relationships. The exceedingly widespread use of temperature in science, engineering and everyday life, however, makes it expedient from a practical viewpoint to include it into the basic quantities. In light engineering the quantities characterizing the subjective perception of light (luminous intensity, illumination, and luminance) are significant. For this reason the use of only energy para-

meters in defining these quantities deprives them of the most important property—a characteristic of their action on our vision. Thus, in applying the cgs system to all physical phenomena, not three, but five of its units should be considered as basic ones.

As mentioned above, at present the International System (SI) has been legally made the most preferable one. This system is a development of the practical system of electrical units, in which two more units—the degree, measuring the thermodynamic temperature, and the candela, measuring luminous intensity have been added to the basic units metre, kilogram, second, and ampere. The definitions of these units will be given below when considering the units relating to the corresponding field of physics.

In 1919 in France there was adopted the metre-ton-second system, in which the unit of mass is a ton (1 000 kilograms). At one time this system was greatly advertized and was even legalized by the relevant standards. It did not become widespread, however, and at present it is practically completely out of use, except for some of its units that have become non-system ones.

1.6. Non-System Units

Notwithstanding certain advantages obtained by using units determined by a system, up to the present wide use is made of various units that do not fit into any of the systems. Many of them cannot be discarded owing to the convenience of using them in definite fields, while others have been retained as a result of historical traditions.

In prerevolutionary Russia there was an old Russian system of weights and measures that in 1924 was replaced by the metric system. The names of the units of this system have been retained at present only in Russian sayings and proverbs, and only the unit of weight *pood* (40 Russian pounds or 16.3805 kg) is sometimes encountered in reports on the production of agricultural goods. Even at present in some countries units have been conservatively retained whose inconvenience not only consists in that they are not constructed according to the decimal system, but also in that one name often hides several units (there are several

miles, gallons, not completely accurately coinciding inches, etc.).

Among the non-system units, decimal multiples and submultiples of units should be set apart into a first group. The names of these units are formed with the aid of the relevant prefixes (deci-, centi-, milli-, deca-, hecto-, kilo-, etc.). A list of these prefixes and the symbols used for them are given in Table 52.

A second group of non-system units is formed by units constructed from the units of a system without following the decimal principle. These include first of all the units of time, minute and hour.

A third group includes, finally, units that have no relation with units of the established systems. They include, in particular, the unit of length inch, the unit of the amount of heat calorie, the units of pressure standard atmosphere and millimetre of mercury, etc.

When considering the units of a quantity in the following chapters, together with the units included in the cgs, SI or mk(force)s systems, we shall give the most widespread non-system units and show their relation to the system units.

CHAPTER TWO

CONVERSION OF UNITS AND DIMENSION FORMULAS

2.1. Dimension Formulas

The existence of different systems gives rise to the problem of converting units of one system into those of another. Obviously, a change in the basic units should lead to a change in the derived ones. Thus, for example, if we take the kilometre as the unit of length instead of the metre, then the unit of velocity will be "kilometre per second", which is 1 000 times greater than the unit "metre per second". If we take the hour as the unit of time and retain the metre as the unit of length, then the unit of velocity will be "metre per hour", which is 1/3 600th of the unit "metre per second". Finally, if we take the kilometre as the unit of length and the hour as the unit of time, the unit of velocity will be "kilometre per hour", equal to $1\ 000/3\ 600 \cong 0.278$ m/s. Thus it can be seen that any change in the basic units correspondingly changes a derived unit.

It is obviously desirable to find such a relationship that would make it possible to determine how the derived unit of a quantity of interest to us changes with a change in each of the basic units. Such a relationship is called *the dimension formula of a unit of the given quantity*.

For acquaintance with how the dimension formulas are constructed and applied for the mutual conversion of units, let us first consider the case when the systems have the same basic quantities and use the same defining relationships. For example, the cgs and SI systems are such systems for mechanical measurements, since only three basic units are used for this purpose from both systems, namely, length, mass and time, and the systems differ from each other only in the dimension of the basic units.

It should be noted that if a derived unit changes n^p times with a change in the unit of length of n times, the given derived unit is said to have the dimension p relative to the unit of length. In the same way, if a derived unit changes proportional to the q-th power of a change in the unit of mass and to the r-th power of a change in the unit of time, then the derived unit is said to have a dimension q relative to the unit of mass and r relative to the unit of time. If the unit of a certain quantity A has the dimensions p, q and r relative to the units of length, mass and time respectively, then this is symbolically written as

$$[A] = L^p M^q T^r \tag{2.1}$$

where the brackets enclosing the symbol of the quantity A denote that we are dealing with the dimension of a unit of this quantity, while the symbols L, M and T are generalized designations of the units of length, time and mass without indicating the concrete magnitude of the unit.

Formula (2.1) can be interpreted to have the meaning that if the ratios between the units of length, mass and time in two systems are equal respectively to L, M and T, then the ratios between the derived units will be $L^p M^q T^r$.

Formula (2.1) is called the *formula of the dimension of a unit of a given quantity*, or, as is frequently said for brevity, the *dimension of a given quantity*. It can be easily seen that a dimension formula can be written only for such quantities whose quantitative characteristic meets the condition of the absolute value of a relative magnitude. It has been found that with any selection of the basic units, the dimension formula of a derived unit is a monomial consisting of the product of the symbols of the basic units to certain powers, and the latter can be positive and negative, integers and fractions*.

In compiling the dimension formulas of derived units we shall use the following theorems.

1. If the numerical value of a quantity C is equal to the product of the numerical values of the quantities A and B,

* Those who desire will find a simple proof of this tenet in the book *Dimensional Analysis* by P. W. Bridgman, New Haven, Yale University Press, 1932.

then the dimension of C is equal to the product of the dimensions of A and B, i.e.

$$[C]=[A][B] \tag{2.2}$$

In other words, if

$$[A]=L^{p_a}M^{q_a}T^{r_a}$$

and

$$[B]=L^{p_b}M^{q_b}T^{r_b}$$

then

$$[C]=L^{p_a+p_b}M^{q_a+q_b}T^{r_a+r_b} \tag{2.3}$$

2. If the numerical value of a quantity C is equal to the quotient of the numerical values of the quantities A and B, then the dimension of C is equal to the quotient of the dimensions of A and B, i.e.

$$[C]=\left[\frac{A}{B}\right]=\frac{[A]}{[B]} \tag{2.4}$$

or

$$[C]=L^{p_a-p_b}M^{q_a-q_b}T^{r_a-r_b} \tag{2.5}$$

3. If the numerical value of a quantity C is equal to the n-th power of the numerical value of the quantity A, then the dimension of C is equal to the n-th power of the dimension of A, i.e.,

$$[C]=[A^n]=[A]^n \tag{2.6}$$

Hence, if

$$[A]=L^pM^qT^r$$

then

$$[C]=L^{pn}M^{qn}T^{rn} \tag{2.7}$$

The proofs of all these theorems are very simple, and for this reason we shall only give the first of them.

If the numerical value of a quantity C is equal to the product of the numerical values of the quantities A and B, this means that when measuring these quantities by means of the units c_1, a_1 and b_1 we get

$$C_1=A_1B_1 \tag{2.8}$$

where

$$C_1 = \frac{C}{c_1}, \quad A_1 = \frac{A}{a_1}, \quad \text{and } B_1 = \frac{B}{b_1} \tag{2.9}$$

When measuring the same quantities by means of the units c_2, a_2 and b_2, we correspondingly get

$$C_2 = A_2 B_2 \tag{2.10}$$

where

$$C_2 = \frac{C}{c_2}, \quad A_2 = \frac{A}{a_2}, \quad \text{and } B_2 = \frac{B}{b_2} \tag{2.11}$$

By dividing Eq. (2.8) by Eq. (2.10) and taking into account Eqs. (2.9) and (2.11), we get

$$\frac{c_2}{c_1} = \frac{a_2}{a_1} \frac{b_2}{b_1} \tag{2.12}$$

If

$$\frac{a_2}{a_1} = L^{p_a} M^{q_a} T^{r_a} \tag{2.13}$$

and

$$\frac{b_2}{b_1} = L^{p_b} M^{q_b} T^{r_b} \tag{2.14}$$

then

$$\frac{c_2}{c_1} = L^{p_a + p_b} M^{q_a + q_b} T^{r_a + r_b} \tag{2.15}$$

Thus the first theorem has been proved. It is quite obvious that the remaining theorems can easily be proved in the same way.

It is important to note the following circumstance. Since the method of constructing a derived unit includes the equation to unity (or to any other arbitrary constant number not depending on the dimensions of the basic units) of the factor of proportionality in the defining relationship, this means that we agree to consider this factor to have a zero dimension or, as is generally said, to be a dimensionless one. It is understood, in addition, that any constant numerical factor obtained in mathematical operations should also be considered as a dimensionless one.

Let us explain the above with examples.

1. The dimension of the area of a square

$$[A_{sq}] = [l]^2 = L^2M^0M^0 \tag{2.16}$$

or omitting here, as will be done in the following, the symbols of the basic units having a zero dimension,

$$[A_{sq}] = L^2 \tag{2.17}$$

2. The dimension of the area of a circle

$$[A_{cir}] = \left[\frac{\pi}{4}\right][D]^2 = L^2 \tag{2.18}$$

since the factor $\pi/4$ is a constant one not depending on the dimensions of the basic units, and therefore a dimensionless one. For this reason the dimension of the area of any geometrical figure, regardless of its shape, will be

$$[A] = L^2 \tag{2.19}$$

3. The dimension of velocity can be determined from the formula for the velocity of uniform motion:

$$[v] = \frac{[l]}{[t]} = LT^{-1} \tag{2.20}$$

4. The dimension of acceleration is determined from the formula for uniformly accelerated motion:

$$[a] = \frac{[v_2 - v_1]}{[t]} = LT^{-2} \tag{2.21}$$

For purposes of illustration let us use the last formula to find how the unit of acceleration will change if we change over from measuring length in metres and time in seconds to kilometres and minutes, respectively. Here the unit of length will increase 1 000 times, and that of time 60 times. According to formula (2.21), the unit of acceleration will change 1 000/60^2 = 10/36 times, i.e., the new unit of acceleration will equal 0.278 of the old one.

5. The dimension of kinetic energy, determined by the formula

$$E_k = \frac{mv^2}{2} \tag{2.22}$$

will obviously be equal (in the SI and cgs systems) to

$$[E_k] = L^2MT^{-2} \tag{2.23}$$

From the latter formula, in particular, it follows that if, in measuring length, we change over from centimetres to metres and in measuring mass from grams to kilograms, and retain the second as the unit of time, the unit of kinetic energy will increase $(100)^2 \times 1\,000 = 10^7$ times.

6. Newton's second law, written in the form

$$Ft = mv_2 - mv_1 \tag{2.24}$$

where the product Ft is called the impulse of the force, and mv the quantity of motion or momentum *, determines the dimension of force:

$$[F] = LMT^{-2} \tag{2.25}$$

In the following, when investigating the units of derived quantities, we shall always use the dimension formulas.

The dimension formula of a derived unit often determines both its name and the symbols used to designate it. For example, the unit of velocity "metre per second" is designated m/s, the unit of area "square metre" m², etc.

2.2. Conversion of Dimension When Using Different Basic Units

If we change the quantities whose units are taken as the basic ones without changing the defining relationships, then the dimension formulas will change correspondingly, for example, when changing over from the SI or cgs system to the mk(force)s system, in which the basic quantities include force instead of mass. Here the transition from one system to the other can be performed if in the dimension formulas we substitute for the dimension of the corresponding basic unit its dimension expressed in the other system. If we designate the dimension of force in the mk(force)s system by the symbol F, then from formula (2.25) we can obtain the dimension of mass in this system:

$$M = [m] = L^{-1}FT^2 \tag{2.26}$$

* The term "momentum" is at present mainly used in theoretical mechanics. The symbol $\overline{p}$ is sometimes used instead of mv. In the theory of relativity, quantum mechanics, atomic and nuclear physics the term "impulse" is generally accepted. The term "quantity of motion" is practically obsolete.

By inserting the dimension of M from this formula into the dimension formulas of various quantities in the SI and cgs systems, we shall obtain the dimensions of these quantities in the mk(force)s system. Thus, for example, the corresponding dimension of kinetic energy is

$$[E_k] = LF \tag{2.27}$$

As an example of reverse conversion the dimension of pressure and mechanical stress can be given, which in the mk(force)s system will be

$$[p] = L^{-2}F \tag{2.28}$$

and, correspondingly, in the SI and cgs systems

$$[p] = L^{-1}MT^{-2} \tag{2.29}$$

2.3. Conversion of Dimensions with Different Defining Relationships

When establishing a derived unit with the aid of a defining relationship, i.e., a mathematical formulation of a definition or a law connecting the given quantity with quantities taken as the basic ones or determined previously, the factor of proportionality in the relationship is assumed to equal unity or some other constant number.

From the viewpoint of construction of the dimension formula, this means that we deprive it of a dimension relative to the basic units or, which is the same, give it a zero dimension. In other words, we agree to consider the factor unchangeable upon any change in the basic units provided that the defining relationship does not change. Should this condition not be observed and another defining relationship be used for determining the derived unit, then the factor of proportionality may change correspondingly. For example, if we use the area of a circle instead of that of a square for determining the unit of area, then, as we have seen (in Sec. 1.4), the factor of proportionality in the formula for the area of a square becomes equal to $4/\pi$ instead of unity, since the factor of proportionality is taken equal to unity in the new defining relationship (the formula for the area of a circle). The transition from square to round

units of area makes it necessary to correspondingly change the factors of proportionality in all formulas relating to the measurement of areas. Here, however, the dimension of area

$$[A] = L^2$$

remains the same, since as a matter of fact in this case also we use the same theorem on the relationship between the area of geometrical figures and their linear dimensions and take the factor of proportionality in a particular expression of this theorem (the formula for the area of a circle) equal to unity.

The above example can be considered, by the way, not as a transition from one defining relationship to another one, but as the replacement of the factor of proportionality, previously taken as unity, by a factor of $4/\pi$ in the formula for the area of a square.

Such a change of the defining relationship is, however, possible that will make the factor of proportionality dimensional, i.e., dependent on the dimensions of the basic units. This can be best illustrated by using the example of establishing the unit of force as a derived unit in systems with length, mass and time as the basic quantities. In the usual determination of the unit of force by means of Newton's second law, we obtain the following dimension of force [see formula (2.25)]

$$[F] = LMT^{-2}$$

If we insert this dimension in the expression for the law of universal gravitation [formula (1-10)], then the following dimension will be obtained for the gravitational constant:

$$[G] = L^3M^{-1}T^{-2} \qquad (2.30)$$

(Here and below we shall revert to the generally used symbol G for the gravitational constant.) The fact that the gravitational constant has a dimension means that its numerical value depends on the selection of the basic units. To find this relationship it should be remembered that a dimension formula shows how a derived unit changes when the basic units are changed. Therefore, by conditionally introducing "the unit of the gravitational constant", we can say on the basis of formula (2.30) that this "unit"

changes in proportion to the cube of the unit of length, inversely proportional to the unit of mass and inversely proportional to the square of the unit of time. Since the numerical value of a quantity upon a change in the units measuring it changes in the reverse proportion [see formula (1.1)], then, consequently, the numerical value of the gravitational constant will be inversely proportional to the cube of the unit of length and directly proportional to the unit of mass and the square of the unit of time. Thus, if the basic units are metre, kilogram and second, the gravitational constant is numerically equal to 6.67×10^{-11}, then upon changing over to the basic units, centimetre, gram and second, its value will become 6.67×10^{-8}.

If for determining the unit of force we use the law of universal gravitation instead of Newton's second law, then we make the gravitational constant dimensionless, i.e., independent of the basic units, but equal to a constant number, for example, unity. The dimension of force will be equal to

$$[f] = L^{-2}M^{2} \tag{2.31}$$

while the inertial constant, which was previously equal to unity and had no dimension, now acquires the dimension

$$[C_i] = L^{-3}MT^{2} \tag{2.32}$$

The change in the dimension of force and the appearance of a dimensional inertial constant with the simultaneous disappearance of a dimensional gravitational constant will lead, naturally, to a different mathematical expression of the laws and definitions in the field of mechanics and to a change in the dimension formulas. For example, the dimension of work, which, as previously, is determined by the product of the force, distance and the cosine of the angle between their directions, will now be not

$$[W] = L^{2}MT^{-2}$$

but

$$[W] = [f]\,[L] = L^{-2}M^{2} \cdot L = L^{-1}M^{2} \tag{2.33}$$

The same dimension of work can be obtained in a different way. If we consecutively deduce the relation between the

work and the change in the kinetic energy* in the new system, we get

$$W = C_i \left(\frac{mv_2^2}{2} - \frac{mv_1^2}{2} \right) \qquad (2.34)$$

Inserting the dimensions of mass, velocity and the inertial constant in the right-hand part, we get

$$[W] = L^{-3}MT \cdot M \cdot (LT^{-1})^2 = L^{-1}M^2 \qquad (2.35)$$

Thus, when different systems differ from each other in the selection of the defining relationships, it should be remembered that the factors of proportionality, which in one system are considered to be dimensionless (and usually equal to unity), acquire a dimension in the other system. Upon transition from one system to another the dimensionless factor should be replaced by one having a dimension, or vice versa, for determining the dimension.

If the number of basic units is reduced, as can be done, for example, if we combine Newton's second law and the law of universal gravitation into one general law similar to Kepler's third law, then both the gravitational and the inertial constants become equal to unity or another dimensionless number, and only the dimensions of length and time will remain in the dimension formulas. The conversion of a dimension from systems with three basic units to a system with two basic units can be performed if in the relevant dimension formulas the dimension of mass is replaced with its expression obtained from the formula combining Newton's second law and the law of universal gravitation. If we write this formula as

$$a = C\frac{m}{r^2} \qquad (2.36)$$

and consider that C is dimensionless (for example, equal to unity), the dimension formula for the unit of mass will be

$$[M] = L^3T^{-2} \qquad (2.37)$$

If this expression is inserted into any of the dimension formulas of force derived both from Newton's second law

* We have used the term "kinetic energy" here for the expression $mv^2/2$, although the actual expression for kinetic energy is $C_i mv^2/2$.

and the law of universal gravitation, the same dimensions will naturally be obtained. Indeed

$$[F] = LMT^{-2} = L^4T^{-4} \qquad (2.38)$$

$$[F] = L^{-2}M^2 = L^4T^{-4} \qquad (2.38a)$$

The same also concerns the dimensions of the units of the other quantities relating to mechanics. Let us give some of them for purposes of illustration. The dimension of work and energy is

$$[W] = L^5T^{-4}$$

The dimension of the impulse and the momentum is

$$[ft] = [mv] = L^4T^{-3}$$

2.4. Determining the Relationship between Units of Different Systems

It is the simplest to convert units of one system into those of another when both systems are constructed on the same defining relationships and the same basic quantities, so that the basic units differ only in magnitude. It follows from the above that since in this case the dimension formula of a derived unit is the same in both cases, it is sufficient to insert in this formula the ratios of the magnitudes of the basic units, which should be given either by definition or experimentally, for example by comparing the standards of the relevant units. In addition to the examples given above let us establish the relationship between two units of force determined on the basis of Newton's second law with the following basic units: centimetre, gram, second, and foot, pound and minute. The relations between the basic units are as follows: 1 foot = 30.48 cm (comparison of standards), 1 pound = 453.6 grams (comparison of standards), and 1 minute = 60 seconds (definition). On the basis of the dimension formula

$$[F] = LMT^{-2}$$

we determine the relation between the units of force

$$\frac{\text{unit of system foot, pound, minute}}{\text{unit of system centimetre, gram, second}} = \frac{30.48\times 453.6}{(60)^2} = 3.84$$

Matters are more complicated when with the same defining relationships, units of different quantities are used as the basic ones, as, for example, in the SI and mkg(force)s systems.

Since at least one of the quantities that is taken as a basic one in one system is a derived one in the other system and vice versa, the relationship between the corresponding units should be established. This can obviously be done only by experiment. When determining the relationships between the units of the SI and mkg(force)s systems, the free falling of a body can be used as such an experiment. Here we shall make use of the fact that in the mkg(force)s system the unit of force is the weight (i.e., the force of attraction to the Earth) of the prototype body—the kilogram—while in the SI system the unit of mass is the mass of the same prototype body. It is general knowledge that any free falling body (in particular, the prototype kilogram) acquires under the action of its weight an acceleration that at each given point of the globe is the same for all bodies, but differs at different points of the globe, increasing from the value of 9.7805 m/s^2 at the equator to 9.8322 m/s^2 at a pole. Measurement of the acceleration of gravity at the place of storage of the prototype kilogram (Sèvres) gave the value 9.80665 m/s^2. This value has been called the *normal* acceleration and has been fixed as a constant value not to be precised*. Whenever the accuracy of measurements or calculations allows an error of over 0.3%, at all points of the Earth's surface the weight of one kilogram can be taken equal to the established technical unit of force—the kilogram(force).

Thus, the result of an experiment consisting in measuring the acceleration of free falling of a body (in our case the prototype kilogram) can be formulated as follows: "a force of 1 kgf imparts to a mass of 1 kg an acceleration of 9.81 m/s^2".

Now let us establish the unit of force in the SI system and the unit of mass in the mk(force)s system, using in both instances Newton's second law. Obviously, in the SI system

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* In the following we shall use only the approximate value 9.81 m/s^2 for the acceleration of gravity.

the unit of force is the force that imparts to a mass of 1 kg an acceleration of 1 m/s^2. This unit of force, as is known (see Sec. 1.3) is called the newton (N). In the mk(force)s system the unit of mass is a mass that under the action of a force of 1 kgf acquires an acceleration of 1 m/s^2. This unit of mass has not been given a special name established by any State Standard. Sometimes it is called the "technical unit of mass" (t.u.m.). USSR State Standard GOST 7664-61 requires this unit to be designated, according to its dimension, i.e., $kgf \cdot s^2/m$. Some time ago professor M. F. Malikov proposed calling this unit the *inerta** (i). Although this name has not only never been legalized anywhere, but has not even come into use, we shall use it in the following owing to its brevity and convenience of designation.

Let us now write the following three equations

$$1\ \text{kgf} = 1\ \text{i} \cdot 1\ \text{m/s}^2 \tag{2.39}$$

$$1\ \text{kgf} = 1\ \text{kg} \cdot 9.81\ \text{m/s}^2 \tag{2.40}$$

$$1\ \text{N} = 1\ \text{kg} \cdot 1\ \text{m/s}^2 \tag{2.41}$$

The first equation expresses the definition of the technical unit of mass, the inerta, the third the definition of the newton, and the second the result of the experiment described above. In the first two equations the same forces impart different accelerations to different masses. Since the accelerations acquired are inversely proportional to the masses, then, as a corollary, we get

$$1\ \text{i} = 9.81\ \text{kg} \tag{2.42}$$

or, conversely,

$$1\ \text{kg} = \frac{1}{9.81}\ \text{i} = 0.102\ \text{i} \tag{2.42a}$$

In the second and third equations different forces impart different accelerations to the same masses. Here, since the forces acting on the bodies are proportional to the accelerations imparted to them, we have

$$1\ \text{kgf} = 9.81\ \text{N} \tag{2.43}$$

* Other names proposed for this unit are the *metric slug*, the *mug*, the *par*, and the *TME*.

or, conversely,

$$1\ \mathrm{N} = \frac{1}{9.81}\ \mathrm{kgf} = 0.102\ \mathrm{kgf} \qquad (2.43a)$$

Mastering of these relationships will help our readers to avoid numerous errors that are usually made during the initial period when solving problems in mechanics.

Since we now have the relationships between the units of mass and force in the SI and mk(force)s systems, it will be simple to establish the relationships between any derived units of these two systems, using the dimension formulas. It will be almost as simple to determine the relationships between units of the cgs and mk(force)s systems, since the former are very simply related to the units of the SI system. Let us give two examples for purposes of illustration.

1. Find the relationship between the units of pressure in the mk(force)s and cgs systems. Let us use dimension formula (2.29) of the unit of pressure in the cgs system

$$[p] = L^{-1}MT^{-2}$$

Since the unit of length in the mk(force)s system (the metre) is 100 times greater than that in the cgs system (the centimetre), the unit of mass in the mk(force)s system (the inerta) is $9.81 \times 1\,000 = 9\,810$ times greater than the unit of mass in the cgs system (the gram), and the unit of time (the second) is the same in both systems, then, according to the dimension formula, the unit of pressure in the mk(force)s system will be $100^{-1} \times 9\,810 = 98.1$ times greater than that in the cgs system. The same result can be obtained if we use dimension formula (2.28) of the unit of pressure in the mk(force)s system

$$[p] = L^{-2}F$$

The ratio of the units of length here, as previously, is 100, while the ratio of the units of force, as the reader himself can easily find, is 9.81×10^5. The ratio of the units of pressure will accordingly be $100^{-2} \times 9.81 \times 10^5 = 98.1$, which corresponds to the result obtained above.

2. Find the relationship between the units of power in the mk(force)s and cgs systems. The dimension of the unit

of power in the cgs system is

$$[P] = L^2MT^{-3} \tag{2.44}$$

By using the known ratios between the units of length and mass, we find the sought relationship

$$100^2 \times 9{,}810 = 9.81 \times 10^7$$

In the same way when using the dimension formula of power in the mk(force)s system

$$[P] = LFT^{-1} \tag{2.45}$$

the ratio of the units will be

$$100 \times 9.81 \times 10^5 = 9.81 \times 10^7$$

Now let us consider the conversion of units in the most complicated case when different defining relationships are used to determine the derived unit in the two systems. Here we shall confine ourselves only to that case, which is of the greatest interest, when the basic quantities in both systems are the same.

Let us have a certain quantity A whose units a_1 and a_2 in two different systems (based on different laws) have the dimensions

$$[A]_1 = L^{p_1}M^{q_1}T^{r_1} \text{ and } [A]_2 = L^{p_2}M^{q_2}T^{r_2}$$

the numbers A_1 and A_2 that express the quantity A in these units being in the following relationship

$$A_1 = CA_2 \tag{2.46}$$

Here C is a factor of proportionality that is now not an abstract quantity, but one depending on the selection of the basic units. The unit by means of which C is "measured" obviously has the dimension

$$[C] = \frac{[A]_1}{[A]_2} = L^{p_1-p_2}M^{q_1-q_2}T^{r_1-r_2} \tag{2.47}$$

Since the numbers measuring a quantity in different units are inversely proportional to these units, we can write

$$a_1 = \frac{1}{C}a_2 \tag{2.48}$$

The ratio of the dimensions of the quantity A in the first and second system gives the dimension of the factor C. Thus, if we know the numerical value of this factor in one system of units, it is possible to determine its numerical value in any other system and thus find the relationship between the corresponding units of the given quantity A. Let us explain what has been said using the example of force that we have considered. When measuring force with the inertial unit, the law of universal gravitation has the form

$$f = G\frac{m_1 m_2}{r^2}$$

If the basic units in both systems are the same, then the expression $m_1 m_2/r^2$ in the right-hand part represents the same force of mutual attraction, but measured in gravitational units (the latter are sometimes called astronomical units). Consequently, upon designating the number that measures force in the inertial system by f_i, and in the gravitational one by f_g, we can write

$$f_i = G f_g \tag{2.49}$$

With a change in the basic units the numbers f_i and f_g will also change, but not to the same extent. For this reason the numerical value of the factor G will change. To determine the nature of this change, let us use the dimension formulas

$$[f]_i = LMT^{-2} \text{ and } [f]_g = L^{-2}M^2$$

whence

$$[G] = \frac{[f]_i}{[f]_g} = L^3M^{-1}T^{-2} \tag{2.50}$$

As we have already seen (in Sec. 2.3), the numerical value of the gravitational constant is inversely proportional to the cube of the unit of length and directly proportional to the unit of mass and the square of the unit of time.

It should be remembered that with the basic units metre, kilogram and second $G = 6.67 \times 10^{-11}$, and with the basic units centimetre, gram and second $G = 6.67 \times 10^{-8}$. Since the relationship between the inertial and the gravitational

units of force (let us designate them φ_i and φ_g) is

$$\varphi_i = \frac{1}{G}\varphi_g \tag{2.51}$$

then, when measuring the mass in kilograms and the distance in metres, we get

$$\varphi_i = \frac{1}{6.67\times 10^{-11}}\varphi_g = 1.5\times 10^{10}\varphi_g \tag{2.51a}$$

and when measuring the mass in grams and the distance in centimetres, correspondingly,

$$\varphi_i = \frac{1}{6.67\times 10^{-8}}\varphi_g = 1.5\times 10^{7}\varphi_g \tag{2.51b}$$

Introducing the designations of the units of force, we can write instead of equations (2.51a) and (2.51b)

$$1\ \mathrm{N} = 1.5\times 10^{10}\ \mathrm{kg^2/cm^2\ units} \tag{2.51c}$$

$$1\ \mathrm{dyn} = 1.5\times 10^{7}\ \mathrm{g^2/cm^2\ units} \tag{2.51d}$$

It is also not very difficult to determine the relationship between units when the dimension of the basic units is different in the two systems. This can be done in the simplest and most illustrative way if we first convert one of the units into a system with the same basic quantities, but with the dimensions of the basic units the same as in the second system.

It goes without saying that all such conversions can be accomplished only on condition that in the system having a dimensional factor of proportionality the numerical value of this factor is known either directly or can be obtained by conversion from another system with the same defining relationships.

2.5. Compilation of Conversion Tables

To avoid the conversion of one set of units into another one in every calculation, it is good practice to compile tables by means of which a quantity measured in one unit can be expressed through any other unit of the same quantity. A special conversion table will be required for each quantity.

The units that are to be converted are arranged at the left-hand side of such a table, and those which they are to be converted to at the top of the table.

Let us take for example the units of length. The corresponding table is given in Appendix 5 at the end of the book (Table 2). The number 39.4 at the intersection of the line "1 m =" and the column "Inch" shows that one metre contains 39.4 inches.

When compiling conversion tables, use is made either of relationships based directly or indirectly on experiment (as, for example, 1 kgf = 9.81 N), or on definition (for example, 1 m = 100 cm), or established by the comparison of standards or prototypes, or, finally, by calculations similar to those given above and based on the use of dimension formulas.

Such conversion tables, compiled for the most important quantities considered and given at the end of the book, may be useful in solving a great diversity of problems. These tables, in addition to the units of different systems, include a number of the most popular non-system units.

2.6. On the So-called Meaning of Dimension Formulas

The examples considered in the previous sections show that the dimension formula of a unit of the same quantity can have different forms depending on the defining relationship used to establish the unit. We consider this tenet to be quite important, since attempts are often encountered in literature to find some "secret meaning" in dimension formulas. Moreover, the widespread and abbreviated expression "dimension of a quantity" used above is often understood literally as some unchangeable property characteristic of the given physical quantity. The possibility of constructing different dimension formulas with a different selection of the defining relationships clearly shows the erraneous nature of such a view.

In this connection it will be appropriate to quote M. Planck: "From this we again see that the dimensions of a physical quantity are not inherent in it, but constitute a conventional property conditioned by the choice of the

system of measurement. If this circumstance had always been properly appreciated, a great number of unfruitful controversies in physical literature, particularly concerning that of the electromagnetic system of measurement, would have been avoided"*, and "the fact that when a definite physical quantity is measured in two different systems of units, it has not only different numerical values, but also different dimensions, has often been interpreted as an inconsistency that demands explanation, and has given rise to the question of the 'real' dimensions of a physical quantity... it is clear that this question has no more sense than inquiring into the 'real' name of an object".**

Naturally, if we remain within the limits of a definite system, then a definite dimension will be retained for the unit of each quantity. In some of the simplest cases the dimension formula will give a notion of how a derived unit has been obtained from the basic ones.

This is natural, since, as we have already mentioned, the principle underlying the construction of derived units reflects the possibility of determining the value of a physical quantity by indirect measurement. For example, the dimension of velocity LT^{-1} shows that to find the velocity the distance and time should be measured, and the relevant numerical values divided. It will be shown below that the dimension of capacitance in the cgs system coincides with that of length. This can be considered as a reflection of the fact that the capacitance of insulated conductors having the same shape is proportional to their linear dimensions. Not many of such examples can be given, however, and in the majority of cases the dimension formula does not give a clear notion of the relation between a given physical quantity and other quantities, in particular, those taken as the basic ones.

Indeed, if we take as an example the dimension formula of such a static quantity as pressure or mechanical stress, which in the SI and cgs systems is

$$[p] = L^{-1}MT^{-2}$$

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* Max Planck, *Introduction to Theoretical Physics*, vol. 1, *General Mechanics*, Sec. 28, London, 1933.

** Ibid, vol. 3, *Electricity and Magnetism*, Sec. 7.

it will hardly be possible to find any physical meaning in the unit of length and the square of the unit of time in the denominator. And, of course, no concrete notions are called forth by the dimension formulas of electrical units in the cgs system, in which the symbols of the dimensions of the basic units are quite frequently in fractional powers.

The very limited significance of dimension formulas can also be seen from the fact that the units of different quantities sometimes have the same dimension. This, of course, should never be interpreted as to mean that these quantities have a common physical nature. Moreover, we shall encounter such quantities among those being considered whose units are dimensionless in a certain system, i.e., do not depend on the selection of the basic units.

A typical example of such quantities is an angle. Although its units can be different (degrees, minutes, parts of a circle, radians), none of them change upon a change in the basic units.

Besides employing dimension formulas for the conversion of units from one system into another and establishing relationships between units, they are used for checking the correctness of formulas obtained by theoretical deduction. The constancy of a dimension formula within the limits of a given system requires that the dimensions in the left-hand and right-hand parts of any equation relating different physical quantities (or, more exactly, the numbers measuring these quantities) be the same. Otherwise when going over from one set of units to another the equation would be violated. For this reason, upon obtaining as a result of reasoning or solution of a problem a formula expressing the relationship between a quantity interesting us and other quantities, the coincidence of the dimensions of the left-hand and right-hand parts of the equation should be checked. If these dimensions do not coincide, it can be said that an error has been made and the equation is not correct.

It should be understood that coincidence of the dimensions is not at all a guarantee that the equation obtained is correct.

2.7. Brief Conclusions on Chapters One and Two

The previous sections set out the principles underlying the construction of systems of basic and derived units and dimension formulas, and also the methods of transferring from one system to another.

For the reader's convenience, the contents of these sections is briefly summarized below in the form of the following conclusions:

1. A measurement is a comparison of the given quantity with another homogeneous quantity taken as a unit.
2. The condition for objective measurement and establishment of units of measurement is the possibility of obtaining the absolute value of relative quantities.
3. The units of all quantities can in principle be selected independently of one another. The existence of indirect measurements together with direct ones, however, makes it possible to relate the units of different quantities to one another.
4. In constructing systems of mutually related units, the units of several quantities are selected independently of the others and of one another. Such units are conventionally called fundamental or basic ones.
5. For all the remaining quantities there are established the so-called derived units, that are related either to the basic units or to one another with the aid of defining relationships that are mathematical expressions of physical laws and the definitions of physical quantities.
6. The symbols of physical quantities in mathematical expressions of physical laws and definitions do not represent the quantities themselves, but are numerical values expressing these quantities in the units selected for measurement.
7. The relationship between a derived unit and the basic ones is determined by the dimension formula (or, in brief, the dimension), which is a monomial formed by the product of the generalized designations of the basic units to different powers.
8. A mathematical expression showing the relationship between different physical quantities should have dimensional homogeneity (the dimension formulas of the left-hand and right-hand parts should be the same). The equa-

tion may have a dimensional factor of proportionality, i.e., one whose numerical value changes when the basic units are changed.

9. A combination of basic and derived units forms a system of units. The latter is constructed as follows:

(a) quantities are selected whose units are taken as the basic ones (such quantities are conditionally called basic ones);

(b) the dimensions of the basic units are established;

(c) a defining relationship is selected for establishing each derived unit;

(d) the factor of proportionality in the defining relationship is equated to unity (or to another constant value) and, consequently, is assumed to be dimensionless.

10. The construction of a system is in principle quite arbitrary, since the number of basic units and the ones selected, and also their dimensions and the selection of the defining relationships are all arbitrary.

11. The number of basic units is connected with the number of dimensional factors in the mathematical expressions of the physical laws. The greater the number of basic units, the greater the number of such factors.

12. With a different selection of the defining relationships, the dimension formula of a unit of the same quantity may be different. Consequently, the dimension is not a certain unchangeable property of a given physical quantity, but depends on the way the system of units has been constructed.

13. The conversion of units from one system into another is accomplished by means of the dimension formulas, for which purpose it is necessary to have the relation between the basic units, which is established according to the method of constructing the systems, namely:

(a) if both systems have been constructed using the same basic quantities and the same defining relationships, then the relationship between the basic units is determined either by comparing their standards or prototypes, or by the conditional definition of the relation existing between the units (for example, a unit of one of the systems is defined as a multiple or a submultiple of a unit of the other system);

(b) if the defining relationships in both systems are the same, but the basic quantities differ, then it is necessary to establish experimentally the relation between the units of a quantity that is a basic one in one of the systems and a derived one in the other;

(c) if the derived units are constructed with the aid of different defining relationships, then it is first necessary to establish the dimension of the factor of proportionality in one of these relationships written in the system in which it is not the defining one, determine experimentally its value at some known values of the basic units and then, by using the dimension formulas, calculate its value at the dimensions of the basic units corresponding to the given system.

14. While it is theoretically possible to construct a system of units in an arbitrary way, practical considerations impose certain limitations on the number of basic units, and on the selection of the basic quantities and the defining relationships. In particular, it is good practice to have a number of basic quantities that is neither too small nor too great.

CHAPTER THREE

ANALYSIS OF DIMENSIONS

3.1. Determining Functional Relationships by Comparing Dimensions

The application of dimension formulas is not exhausted by the conversion of units and the checking of whether the formulas are correct. If it is known in advance what physical quantities participate in the process being investigated, it is often possible to establish the nature of the relationship connecting the given quantities by comparing the dimensions. In many branches of physics and related sciences—heat engineering, fluid mechanics, etc.—such a method, referred to as the analysis of dimensions, has come into considerable favour. It is especially fruitful when direct determination of a law being sought either encounters considerable mathematical difficulties or requires a knowledge of such details of a process that are unknown beforehand. In essence, analysis of dimensions is based on the same requirement that the relationship between physical quantities be independent of the selection of units which is equivalent to the requirement of coincidence of the dimensions in both parts of equations. While in many instances this does make it possible to rapidly establish the nature of the relationship being sought, the analysis of dimensions is not at all an all-powerful method, and sometimes its possibilities are found to be quite limited.

The objects of the present volume do not include a detailed consideration of the methods and applications of the analysis of dimensions, which special books are devoted to. We shall limit ourselves only to a brief acquaintance with how dimension formulas can be used to solve practical problems, for which purpose several of the simplest typical examples will be considered below.

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1. A weight having a mass of m is suspended from a spring (Fig. 2). Upon elongation of the spring by h there appears an elastic force equal (in absolute value) to f that tends to return the spring to its initial position. Besides the force f, no other forces act on the weight. Find the time t it takes the weight to return to its initial position.

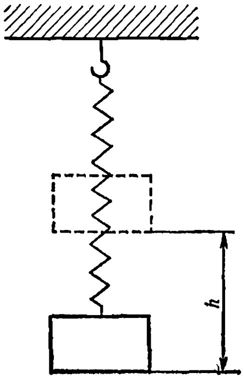

Fig. 2

To solve this problem, it is necessary to represent time as a certain function of the known quantities m, h and f. Although this function may in principle have different forms, certain quite definite considerations can be said about it. Let us assume that this function includes some trigonometrical, exponential or other non-algebraic functions. It is obvious that only dimensionless quantities can be the arguments of the latter. It can be easily seen that in a system of units L, M and T no dimensionless combination can be formed from the quantities m, h and f, whose dimensions are correspondingly M, L and LMT^{-2}, since T is included only in the dimension of force, and for this reason force cannot be included in such a combination, while h and m, naturally, cannot give a dimensionless combination. Thus the only possible kind of relationship between t and h, m and f is an algebraic function. It seems natural to seek this function in the form

$$t = Xf^p h^q m^r \tag{3.1}$$

where X is an unknown dimensionless factor of proportionality, and p, q and r are unknown exponents. Let us equate the dimension formulas of the left- and right-hand parts of equation (3.1):

$$T = L^p M^p T^{-2p} L^q M^r \tag{3.2}$$

Equation (3.2) will be an invariant one with respect to the dimension of the basic units (i.e., it will remain in

force upon an increase or a reduction in the value of the basic units) if the exponents of the basic units in the left- and right-hand parts are equal. On the basis of this condition, we get the following equations for the exponents:

$$0 = p + q;\ 0 = p + r;\ \text{and}\ 1 = -2p \tag{3.3}$$

whence

$$p = -\frac{1}{2};\ q = \frac{1}{2};\ \text{and}\ r = \frac{1}{2} \tag{3.4}$$

Accordingly,

$$t = X\sqrt{\frac{mh}{f}} \tag{3.5}$$

Naturally, the above analysis does not allow us to judge of the value of the factor X. If the force f is proportional to h (as is the case for elastic forces), then

$$f = C_e h \tag{3.6}$$

where C_e is the coefficient of elasticity of the spring, and we can write the equation

$$t = X\sqrt{\frac{m}{C_e}} \tag{3.7}$$

so that the time does not depend on the elongation h. Exact solution of this problem based on the application of the laws of mechanics leads to the same equation (3.7), but with a definite factor X equal to $\pi/2$*.

2. An ideal (non-viscous) liquid with a density of ρ is poured into a cylindrical vessel with a cross-sectional area of A_1 to a level at a height h from the bottom (Fig. 3). The bottom of the vessel has an orifice with the cross-sectional area A_2. Find the time t it will take the liquid to flow out.

Since the liquid flows out under the action of the force of gravity, it is natural to assume that among the quantities determining the process there should be the acceleration of gravity. It is possible here in principle that the relationship

* It can be easily seen that for the force of gravity (the force is proportional to the mass, $f = mg$) formula (3.5) transforms into the formula for the duration of free falling $X\sqrt{h/g}$, where X is a dimensionless factor whose numerical value, as is known, is equal to $\sqrt{2}$.

being sought contains a transcendental function including the quantities h, A_1 and A_2 in the argument (ρ and g cannot be in this argument owing to the considerations given above). Nevertheless we shall here also try to represent the time being sought in the form of an exponential monomial:

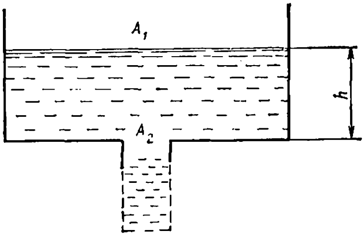

Fig. 3

$$t = X\rho^p g^q h^r A_1^k A_2^l \tag{3.8}$$

where, as above, X is a dimensionless and non-determinable factor of proportionality, and p, q, r, k, and l are exponents to be determined. Let us compile a dimension equation

$$T = M^p L^{-3p} L^q T^{-2q} L^r L^{2k} L^{2l} \tag{3.9}$$

whence, by equating the exponents of the lef- and right-hand parts, we get the following simultaneous equations

$$\left.\begin{aligned} 0 &= -3p + q + r + 2(k+l) \\ 0 &= p \\ 1 &= -2q \end{aligned}\right\} \tag{3.10}$$

We have only three equations for determining five exponents. Two of them, however, are determined directly:

$$p = 0 \text{ and } q = -\frac{1}{2} \tag{3.11}$$

This is already of certain interest, since it shows that the duration of outflow does not depend on the density of the liquid and is inversely proportional to the square root of the acceleration of gravity. For determining the remaining exponents it is essential to have either additional data or make assumptions based on our ideas of how the process goes on. Let us assume that the absolute velocity of the liquid flowing through the orifice does not depend on its cross section. Thus the duration of outflow should be inversely proportional to A_2. At the same time the duration of outflow from the same initial level h should be proportional to the total amount of liquid and, consequently, to

the section A_1. This gives the values of 1 and -1 for the exponents k and l. With such an assumption we immediately determine the exponent $r = \frac{1}{2}$, and the **duration** of outflow can be expressed as

$$t = X\sqrt{\frac{h}{g}}\,\frac{A_1}{A_2} \tag{3.12}$$

With respect to the factor X, an analysis of the dimension, as in the previous example, does not make it possible to determine it. Calculations show that this factor is equal to $\sqrt{2}$.

The exponent r can also be found in another way. Since the initial condition of the problem says nothing on the shape of the orifice and that of the cross section of the vessel, the unit of area can be related to the basic ones, not making it dependent on the unit of length. In this case instead of Eq. (3.9) we should write

$$T = M^p L^{-3p} L^q T^{-2q} L^r A^k A^l \tag{3.13}$$

where A is the symbol for the dimension of area.

From this equation, in addition to the conditions that $p=0$ and $q = -\frac{1}{2}$, we get

$$k = -l \text{ and } r = \frac{1}{2} \tag{3.14}$$

Thus, we have as the solution of the problem

$$t = X\sqrt{\frac{h}{g}}\,\varphi\left(\frac{A_2}{A_1}\right) \tag{3.15}$$

where, in contrast to Eq. (3.12), however, $\varphi(A_2/A_1)$ is an unknown function of the ratio A_2/A_1.

It is easy to see that in comparison with the original state of affairs, when a special assumption was required, we were able to achieve greater definity of the solution owing to the introduction of an additional basic unit.

3. A constant force F acts over a path of length h on a body with a mass m. Find the speed the body acquires at the end of this path.

Similar to the previous examples, we write the velocity as

$$v = XF^p m^q h^r \tag{3.16}$$

with the unknown factor X and the exponents p, q and r.
The dimension formula will be

$$LT^{-1} = L^{p}M^{p}T^{-2p}M^{q}L^{r} \tag{3.17}$$

Upon comparing the exponents, we easily get

$$p = \frac{1}{2}; \quad q = -\frac{1}{2}; \text{ and } r = \frac{1}{2} \tag{3.18}$$

whence

$$v = X\sqrt{\frac{fh}{m}} \tag{3.19}$$

The solution of this problem on the basis of the law of conservation of energy gives, as is known,

$$v = \sqrt{\frac{2fh}{m}} \tag{3.19a}$$

i.e., $X = \sqrt{2}$.

Let us approach the same problem in a somewhat different way. Let us try to find the velocity by means of the same analysis of dimensions, but in a system of units in which the unit of force is determined not from Newton's second law, but from the law of universal gravitation. In this system the dimension of force is

$$[f] = L^{-2}M^{2} \tag{3.20}$$

The dimension equation based on Eq. (3.16) will be

$$LT^{-1} = L^{-2p}M^{2p}M^{q}L^{r} \tag{3.21}$$

We have arrived at an absurd contradictory result. In the left-hand part the dimension of time has the exponent —1, while in the right-hand part time is in general absent. What is the reason for this contradiction? Upon considering the essence of the problem, we see that the main law determining the given process—acceleration under the action of an external force—i.e., Newton's second law, has dropped out of consideration. This is important in the respect that in the system of units which we have adopted, the expression for Newton's second law should contain an inertial constant whose dimension is

$$[C_i] = L^{-2}MT^{2}$$

Upon introducing into the equation of dimensions the dimension of the inertial constant, we obtain simultaneous equations, but we shall have only three of them for determining four exponents. Indeed, if instead of Eq. (3.16) we write

$$v = X f^p m^q h^r C_i^k \tag{3.22}$$

we get the simultaneous equations

$$\left.\begin{aligned} 1 &= -2p + r - 3k \\ 0 &= 2p + q + k \\ -1 &= 2k \end{aligned}\right\} \tag{3.23}$$

from which only $k = -\frac{1}{2}$ can be directly determined, so that in the relationship being sought one of the exponents remains unknown, and it can be written, for example, as follows:

$$v = X f^p \left(\frac{h}{m}\right)^{2p-\frac{1}{2}} C_i^{-\frac{1}{2}} = X \left(\frac{fh^2}{m^2}\right)^p \left(\frac{C_i h}{m}\right)^{-\frac{1}{2}} \tag{3.24}$$

The problem can be made completely definite if we introduce another basic unit, namely the unit of force. If we designate its dimension, as previously, by f, the dimension of the inertial constant will be

$$[C_i] = L^{-1} M^{-1} T^2 F \tag{3.25}$$

We can now write the relationship being sought as

$$v = X f^p m^q h^r C_i^k \tag{3.26}$$

and obtain the following simultaneous equations for the exponents:

$$\left.\begin{aligned} 1 &= r - k \\ 0 &= q - k \\ -1 &= 2k \\ 0 &= p + k \end{aligned}\right\} \tag{3.27}$$

whence it is quite simple to determine all the exponents:

$$p = r = \frac{1}{2} \quad \text{and} \quad q = k = -\frac{1}{2}$$

and, consequently,

$$v = X\sqrt{\frac{fh}{mC_i}} \tag{3.28}$$

It is easy to see that if, on the contrary, we reduce the number of basic units, using for the solution of the problem a system with two basic units—length and time—in which the dimensions of force and mass are, respectively,

$$[f] = L^4T^{-4} \tag{3.29}$$

$$[M] = L^3T^{-2} \tag{3.30}$$

then the problem will also become indefinite. Indeed, the equation of dimensions will now be

$$LT^{-1} = L^{4p}T^{-4p}L^{3q}T^{-2q}L^r \tag{3.31}$$

and we obtain only two equations for finding the exponents:

$$\left.\begin{aligned} 1 &= 4p + 3q + r \\ -1 &= 4p - 2q \end{aligned}\right\} \tag{3.32}$$

The solution of the problem again requires additional assumptions that are not quite obvious, notwithstanding the simplicity of the problem itself. Equations (3.32) are simultaneous ones and are satisfied if we substitute for the exponents their values from Eq. (3.18).

3.2. The Π-Theorem and the Method of Similarity

A consideration of the examples given above leads us to the conclusion that the analysis of dimensions cannot be a universal method making it possible to automatically find the relationships between physical quantities participating in a process that are of interest to us. The use of analysis of dimensions frequently requires a suitable selection of the system of units, consideration of the dimensional factors that may enter the expressions for the laws governing the given process, or the definitions of the physical quantities. Additional assumptions are often required that have to be chosen by intuition, and so on.

The examples also show that the smaller the number of basic quantities and the greater the number of parameters participating in a process (including the dimensional fac-

tors), the more incomplete will be the system of equations that can be compiled for finding the exponents of the symbols of the quantities entering the relationship being sought. It is also possible that the dimension equations will lead to an unsolvable system of equations for the exponents in the dimension formulas. As we have seen, this indicates that a quantity essential for solving the problem was not taken into consideration. A dimensional factor may also be such a quantity.

The so-called Π-theorem, whose proof can be found in the cited book by Bridgman and in a book by Sedov*, can render appreciable assistance in analysing dimensions. According to this theorem, if the functional relationship between n physical quantities satisfies the condition of invariance relative to the magnitude of the basic units, and the number of basic units is k, then $n - k$ dimensionless combinations of the quantities can be compiled. The smaller this difference, the more definite will the solution of the problem be. When $n - k = 1$ the problem becomes the most definite and, as a rule, single-valued. By separating the quantity whose relationship to the remaining ones we want to determine from among the total number of quantities, we can express the relationship being sought in the form of an explicit function.

Let us illustrate the above with examples, using for this purpose the ones considered in the previous section.

In the first example (the returning of a weight pulled back by a spring to its initial position) four quantities are related, namely, the mass of the weight, the force of tension of the spring, its elongation and the duration of returning to the initial position. According to the Π-theorem, with three basic quantities—length, mass and time—one dimensionless combination can be formed from four quantities. Accordingly, the relationship between these quantities can be written in the form of the function

$$\varphi\,(f^p h^q m^r t^k) = \text{const} \tag{3.33}$$

where the argument of the function is dimensionless, and the constant quantity forming the right-hand part also has

* L. I. Sedov, *Metody podobiya i razmernosti v mekhanike* (Methods of Similarity and Dimensions in Mechanics), Nauka, Moscow, 1967.

no dimension. In the argument all the exponents, while retaining its dimensionless nature, can be changed by the same number of times, as a result of which one of them can be made to equal unity. It is most convenient to do this, obviously, for the quantity being sought, in the given case the time, so that, equating the exponent k to unity, we get

$$[f^p h^q m^r t] = 1 \tag{3.34}$$

Equation (3.34) is equivalent to equation (3.2), the only difference being that all the exponents have the reverse signs.

Since the basic units have been selected, combined units may be taken as such provided that their dimensions will be independent. For this reason the number of quantities whose dimensions are mutually independent can be taken instead of the number of basic units in the formulation of the Π-theorem. This can be illustrated by returning to the second example (on the outflow of a liquid from a cylindrical vessel). In this example we sought the relationship between the following quantities: the duration of outflow t, the density of the liquid ρ, the acceleration of gravity g, the height of the level h, and the cross-sectional areas A_1 and A_2. Among the dimensions of these quantities T, $L^{-3}M$, LT^{-2}, L, L^2 and L^2 there are three independent ones, namely, T, $L^{-3}M$ and L. Thus three dimensionless combinations can be compiled from the quantities listed above:

$$\frac{h}{gt^2}, \quad \rho^0; \text{ and } \frac{A_2}{A_1} \tag{3.35}$$

By separating the time t from the combination h/gt^2, we can write it as follows:

$$t = \sqrt{\frac{h}{g}}\, \varphi\left(\frac{A_2}{A_1}\right) \tag{3.36}$$

which is what we previously obtained.

Let us consider another example and find the velocity v with which a ball sinks in a viscous liquid. The diameter of the ball d, its density ρ_1, the density of the liquid ρ_2 and its viscosity η are given. Obviously, the acceleration of gravity is also among the quantities determining the process. Thus, to solve the problem we have six quantities with three basic units, which makes it possible to

compile three dimensionless combinations. As we have already seen, the problem becomes the more definite, the smaller the difference between the number of quantities defining a phenomenon and the number of basic units. The present problem can be made more definite if we introduce at least one additional basic unit, preferably the unit of force. The quantities included in the problem will have the following dimensions:

$$[v] = LT^{-1}; \ [d] = L; \ [\rho_1] = [\rho_2] = L^{-3}M;$$
$$[\eta] = L^{-2}TF; \text{ and } [g] = FM^{-1}$$

The explanation of our writing the dimension FM^{-1} for acceleration instead of LT^{-2} is that in the latter case we would have to introduce another dimensional quantity—the inertial constant. By writing the dimension of acceleration as FM^{-1}, we retain the formula of Newton's second law $f = ma$.

Now we can compile only two dimensionless combinations. One of them, similar to the examples previously considered, will obviously be the ratio ρ_2/ρ_1. When compiling the equations for the exponents of the remaining quantities, we easily obtain a second combination including, in particular, any of the densities, for example ρ_1, namely, $v\eta\rho_1^{-1}d^{-2}g^{-1}$. Hence for the velocity of sinking being sought we get

$$v = X \frac{d^2\rho_1 g}{\eta} \varphi \left(\frac{\rho_2}{\rho_1}\right) \tag{3.37}$$

The function $\varphi(\rho_2/\rho_1)$ is not determined by the data of the problem. Naturally, the problem would be still more indefinite if we retained only three basic units. It is interesting to note that an almost identical problem on the velocity with which an air bubble (whose density can be neglected) will rise to the surface of a liquid becomes quite definite, since the number of quantities involved is less by one. It is simple to show that here the dimensionless combination has the form

$$\frac{v\eta}{d^2\rho_2 g}$$

whence the velocity with which the bubble rises in the liquid is

$$v = Xd^2\eta^{-1}\rho_2 g \tag{3.38}$$

A comparison of equations (3.38) and (3.37) shows that the function $\varphi(\rho_2/\rho_1)$ has the form

$$\varphi\left(\frac{\rho_2}{\rho_1}\right) = 1 - \frac{\rho_2}{\rho_1} \tag{3.39}$$

so that equation (3.37) becomes

$$v = X\frac{d^2 g}{\eta}(\rho_1 - \rho_2) \tag{3.40}$$

Theoretical calculations give a value of X of 1/18.

It is easy to see that formula (3.40) describes all the cases of motion of a ball in a viscous liquid, both when $\rho_1 > \rho_2$ and when $\rho_1 < \rho_2$, up to $\rho_1 = 0$, since v can assume both positive and negative values.

The examples given above show once more that when employing analysis of dimensions it is necessary, together with sufficiently obvious procedures, to use intuition not only when determining the quantities of significance for the given specific problem, but also when selecting the basic units and even when writing down the dimensions. Thus, in the last example it was not obvious that the dimension of the acceleration of gravity should be written FM^{-1}, and not LT^{-2}*.

It can be noted that the Π-theorem in itself adds nothing new to the method of employing the analysis of dimensions described above, although it often does make it possible to conduct the analysis in a more convenient way and give its results in different forms depending on the parameters we are interested in. Its main purpose, however, is that it is convenient to introduce the so-called dimensionless similarity criteria with its aid.

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* It is not difficult to see that the reason for this is that in the second case we would have to introduce another dimensional quantity—the inertial constant. By writing the dimension of acceleration in the form FM^{-1}, we retain the formula of Newton's second law $f = ma$.

In principle any of the dimensionless combinations of quantities determining the phenomenon being investigated may be such a criterion. If in such a combination the values of the quantities forming it are so changed that the combination itself does not change, its numerical value will remain constant even when the dimension of the basic units is changed. Consequently, when the remaining quantities are retained, the quantity being sought also remains unchanged. Thus, for example, in the problem on the duration of outflow of the liquid the time is a function of the dimensional ratio h/g, the dimensionless ratio A_2/A_1 and, it can also be said, the dimensionless quantity ρ^0. The latter simply means that the duration of outflow does not depend on the density of the liquid. In the given instance the ratio A_2/A_1 should be considered as the criterion of similarity. Should the area of the vessel cross section A_1 and that of the orifice A_2 be changed by the same number of times, then with a constant value of h (and, of course, of g), the time of outflow will not change.

The introduction of criteria of similarity was found to be especially convenient when sufficient information was not available for complete description of a phenomenon, or when the strict solution of a problem involved great mathematical difficulties.

The first criterion by means of which important theoretical results were obtained relating to the flow of a real (viscous) fluid was introduced by O. Reynolds and bears his name. This criterion, or Reynolds number Re, is equal to

$$\mathrm{Re} = \frac{vD\rho}{\eta} \tag{3.41}$$

where v = velocity of fluid
D = diameter of pipe
ρ = density of fluid
η = viscosity.

The latter, as will be shown below, has the dimension $L^{-1}MT^{-1}$ and, consequently, Re is actually a dimensionless quantity. With a given value of Re the nature of the flow of different liquids in different pipes with different velocities has been found to be the same; the distribution of pressures, velocities, etc. is identical. It has been established

experimentally that when the value of Reynolds number Re reaches 2200 (the so-called critical Reynolds number), the regular laminary flow of a liquid becomes chaotic—turbulent.

The introduction of similarity criteria has been quite helpful in solving a variety of problems in aero- and hydrodynamics, heat transfer, etc. Of especial significance is the fact that the method of similarity can be used to study various phenomena on models. Thus, for instance, Reynolds number (which is applicable not only to the flow of fluids in pipes, but also to the flow of a fluid around bodies submerged in it) makes it possible to investigate the resistance of bodies in a stream of fluid if the bodies are replaced by geometrically similar models of smaller dimensions and the velocity of flow is increased correspondingly.

The similarity criteria in the way they are formed are dimensionless only with a certain selection of the defining relationships, and should the latter be changed, the dimensions of the units entering the expression of the given criterion will also change, and it will acquire a definite dimension. It can be shown, for instance, that when the inertial unit of force is replaced by the gravitational one, Reynolds number acquires the dimension

$$[\mathrm{Re}] = L^3M^{-1}T^{-2} \tag{3.42}$$

It can be easily seen that the criterion can again be made dimensionless if we introduce into it the inertial constant whose dimension, as we know, is equal to $L^{-3}MT^2$.

In conclusion it should be noted that the compiling of dimensionless combinations is also useful when a problem is solved without any great difficulty in the usual way. By so transforming the solution that the quantity being determined is represented as a function of a number of quantities of which at least a part can be collected into dimensionless combinations, it is possible to get an expression convenient for analysis and generalizations.

CHAPTER FOUR

UNITS OF GEOMETRICAL AND MECHANICAL QUANTITIES

4.1. Introduction

For constructing the units of geometrical quantities, only the unit of length is required of all the basic units, the metre in the SI and mk(force)s systems and the centimetre in the cgs system. In kinematics a second basic unit, the unit of time—the second, the same in all the systems, is added to the unit of length. Finally, in dealing with dynamics a third basic unit is introduced, the unit of mass kilogram or gram in the SI and cgs systems, respectively, and the unit of force the kilogram (force) in the mk(force)s system. All these units were given previously, and we shall not consider them here.

In the following sections of this chapter all the most important geometrical and mechanical units will be considered—their formation, definition, determination and dimension formulas in the SI and cgs systems (i.e., with respect to the units L, M and T). The dimension formulas in the mk(force)s system (L, F and T) are given in the summary table (Appendix 5, Table 1) of geometrical and mechanical units relating to the SI, cgs and mk(force)s systems. For each quantity the table gives its name, symbol, the formula used to determine it, the dimension formulas in the SI, cgs and mk(force)s systems, and the symbols of the corresponding units in all three systems. The basic units of each system are given in the table in bold-face type.

Some of the most widespread non-system units of the corresponding quantities will also be given in the following sections,

4.2. Geometrical Units

In addition to the basic system units metre and centimetre, a number of decimal multiples and submultiples of these units are used, of which the following are in the greatest favour:

Kilometre:	1 km = 1000 m = 10^5 cm
Decimetre:	1 dm = 0.1 m = 10 cm
Millimetre:	1 mm = 10^{-3} m = 0.1 cm
Micron:	1 μ = 10^{-6} m = 10^{-4} cm = 10^{-3} mm
Nanometre:	1 nm = 10^{-9} m = 10^{-7} cm = 10^{-6} mm = 10^{-3} μ
Angstrom:	1 Å = 10^{-10} m = 10^{-8} cm = 10^{-7} mm = 10^{-4} μ

Two of these units require additional remarks. The micron is generally designated as written above, i.e. μ. In connection with the introduction of the International System it was proposed to call it micrometre and designate it μm. The nanometre, equal to 10^{-9} m, was previously called millimicron and designated mμ.

In X-ray spectroscopy and X-ray diffraction analysis, a unit of length called the X-unit and designated XU is used. This unit was first introduced as 10^{-3} Å (or 10^{-11} cm), owing to which it was the same as a milliangstrom. Careful comparison, however, showed a certain discrepancy between the X-unit, which is determined with high accuracy in X-ray spectroscopy, and the milliangstrom. Since for a number of years all wavelengths and lattice constants in X-ray diffraction analysis were measured in X-units, and the latter have been used in numerous tables, it was found expedient to retain the X-unit as an independent unit of length

$$1\ \mathrm{XU} = 1.00206 \times 10^{-3}\,\text{Å}$$

The number of units of length of a non-metric origin is exceedingly great and, probably, cannot be counted, since in each country at various times different units were introduced, sometimes not related to one another in any way, and the same name could be given to units of different sizes. Below only some of these units will be given, the selection being determined by the fact that either they are used to a more or less considerable extent, or are often mentioned in literature.

In engineering, mainly in machine-building, wide use up to the present time is made of the *inch*

$$1'' = 2.54 \text{ cm} = 0.254 \text{ m}$$

In navigation the *international nautical mile* is used, which is equal to one angular minute (see below) of a meridian, i.e., 1852 m. One tenth of a nautical mile (185.2 m) is called a *cable* or *cable-length*.

In astronomy the following special units of length are used:

— the *parsec* (pc) is the distance from which half the diameter of the Earth's orbit is seen at an angle of one angular second (see below). One parsec is equal to 3.084×10^{13} km. In addition to the parsec its multiples are frequently used—the *kiloparsec* (kpc) and the *megaparsec* (Mpc);

— the *astronomical unit of length* (AU) is the mean distance from the Earth to the Sun, equal to 1.496×10^8 km (a more accurate value is 1.495993×10^8 km);

— the *light year* (encountered mainly in popular science literature) is the distance covered by light in one year, equal to 9.4605×10^{12} km.

In English-speaking countries the following main units of length are in use:

inch:	$1'' = 2.54$ cm
foot:	$1 \text{ ft} = 12'' = 0.3048$ m
yard:	$1 \text{ yd} = 3 \text{ ft} = 0.9144$ m
mile:	$1 \text{ mile} = 5{,}280 \text{ ft} = 1.609$ km
mil:	$1 \text{ mil} = 0.001'' = 2.54\ \mu$

Area. In all systems the unit of area is the area of a square whose side is equal to a unit of length. From the formula

$$A = l^2 \tag{4.1}$$

we obtain the dimension

$$[A] = L^2 \tag{4.2}$$

The system units of area are the *square metre* in the SI and mk(force)s systems, and the *square centimetre* in the cgs system:

$$1 \text{ m}^2 = 10^4 \text{ cm}^2$$

Formula (4.1) serves as the basis for constructing non-system units of area, among which the ones in greatest use are 1 $km^2 = 10^6$ m^2, 1 $dm^2 = 10^{-2}$ m^2, 1 $mm^2 = 10^{-2}$ cm^2, and 1 $in^2 = 6.4516$ cm^2.

A unit of area of 100 m^2 is called the *are*, and 100 ares equal one *hectare* (ha), a generally used unit for measuring land area:

$$1 \text{ ha} = 10^2 \text{ ares} = 10^4 \text{ m}^2 = 10^{-2} \text{ km}^2$$

In English-speaking countries some units of area are 1 $in^2 = 6.4516$ cm^2, 1 $ft^2 = 0.09290$ m^2, 1 $yd^2 = 0.8361$ m, 1 acre $= 4{,}046.86$ m^2, and 1 $mile^2 = 2.590$ km^2.

Volume. The unit of volume in all systems is the volume of a cube with an edge equal to a unit of length

$$V = l^3 \tag{4.3}$$

Correspondingly the dimension is

$$[V] = L^3 \tag{4.4}$$

In the SI and mk(force)s systems the unit of volume is the *cubic metre* (m^3), and in the cgs system the *cubic centimetre* (cm^3 or cc):

$$1 \text{ cm}^3 = 10^{-6} \text{ m}^3$$

Among other units are

$$1 \text{ dm}^3 = 10^{-3} \text{ m}^3 = 10^3 \text{ cm}^3$$

and

$$1 \text{ in}^3 = 16.384 \text{ cm}^3$$

The *litre* (l), which is frequently called "a unit of capacity", was previously defined as the volume occupied by one kilogram of water at 4°C, equal to 1.000028 dm^3.

In 1964 the litre was equated to one cubic decimetre: 1 l = 1 dm^3.

Angle. In all systems of units an angle is defined as the ratio of the length of an arc to its radius. According to this definition a unit of angle is an angle the length of whose arc is equal to a unit of length with the radius also equal to the unit of length. Since according to this definition the angle

$$\varphi = \frac{l}{r} \tag{4.5}$$

where l is the length of the arc and r the radius, then it is not difficult to see that angle is a quantity with a zero dimension with respect to all the basic quantities, in other words, its unit does not depend on the dimension of the basic units. This universal unit of angle is named the *radian* (rad).

The circumstance that the unit of angle in all three systems has no dimension is often absolutely erroneously interpreted in the sense that angle is an abstract quantity. Actually angle is a full-fledged geometrical quantity. It can be directly measured with the aid of an arbitrary angular measure—a unit of angle, which is sometimes even taken as a basic unit with its own dimension (designated Ω). The absence of a dimension of angle in the SI, cgs and mk(force)s systems only means that with the defining relationship (4.5) accepted in these systems the unit of angle is the same regardless of the magnitude of the basic units. This also makes it possible to easily introduce independent non-system units of angle, namely, the revolution (rev) equal to 2π radians and the degree (deg or °) forming 1/360th of a revolution; the degree is divided into 60 minutes

$$1^\circ = 60'$$

and a minute is divided into 60 seconds

$$1' = 60''$$

In addition, a right angle is used (designated 1D or 1 ∟) equal to 90°, or $\frac{\pi}{2}$ rad, or $\frac{1}{4}$ rev. A right angle is divided into 100 parts, each of which is called a gon (1^g):

$$1^g = 0.01\text{D} = 0.9^\circ = 0.0157 \text{ rad}$$

One gon consists of 100 metric minutes (1^c) and 10^4 metric seconds (1^{cc}):

$$1^g = 10^{2c} = 10^{4cc}$$

It follows from the above that

$$1 \text{ rad} = 57^\circ 17' 45'' = 57.296^\circ$$

or

$$1^\circ = 0.017453 \text{ rad}$$

Figure 4 shows angles of 1° (AOB), 1 rad (AOC) and $\frac{\pi}{2}$ (AOD). For purposes of illustration it may be indicated that a length 1 mm long is seen at an angle of 1′ from a distance of 3.44 m, and at an angle of 1″ from a distance of 206 m.

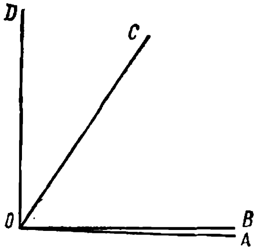

Fig. 4

Solid angle. Before determining a unit of solid angle, let us consider in greater detail the concept of solid angle itself, since it is frequently not completely understood. Let us take a sphere on which a certain closed line is drawn (Fig. 5). If all the points on this line are connected to the centre of the sphere, a cone* is formed enclosing a certain part of space. The cone will be the wider or, in other words,

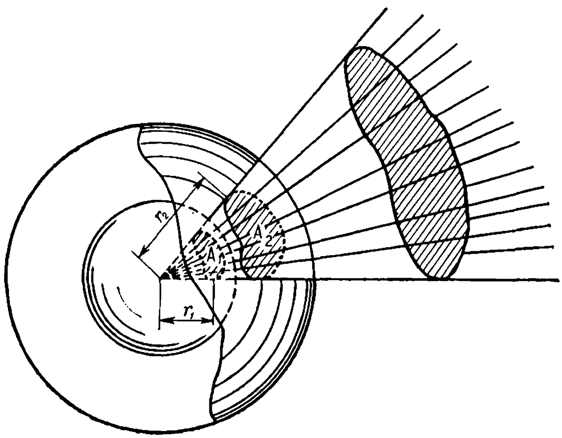

Fig. 5

its flare or divergence will be the greater, the larger the part of the surface of the sphere that is enclosed by the

* A cone in the broad meaning of the term denotes any figure formed by the motion of a straight line with one of its ends fixed and any point on it moving along a closed line.

line. If we now construct from the same centre a number of spheres having different radii, then the cone that we obtained will cut sections out of them that are similar to the one used to construct the cone. The areas of these sections, as is obvious from simple geometrical considerations, will be proportional to the squares of the radii of the spheres which they were cut out from. For this reason the ratio of the area of each of them to the square of the corresponding radius will remain constant regardless of the radius of the sphere and will be the greater, the larger is the flare of the cone. This ratio of the area cut out by a cone on a sphere to the square of its radius is taken in all three systems [SI, cgs and mk(force)s] as a measure of solid angle. Thus, the solid angle Ω is determined by the formula

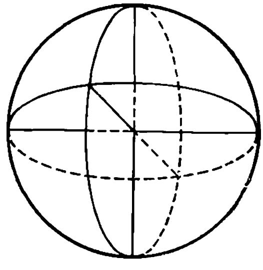

Fig. 6

$$\Omega = \frac{A}{r^2} \tag{4.6}$$

A complete sphere forms a solid angle equal to $\Omega = \frac{4\pi r^2}{r^2} = 4\pi$.

Since when three mutually perpendicular planes intersect a sphere through its centre, it is divided into eight right angles (Fig. 6), the magnitude of each right angle will be $\frac{4\pi}{8} = \frac{\pi}{2}$, the same as that of a right angle on a plane.

It follows from the definition that a solid angle, like an angle on a plane, is a quantity having no dimension. For this reason the unit of solid angle accepted in all systems is the *steradian* (sr), defined as the solid angle subtended at the centre of a sphere by an area on its surface numerically equal to the square of the radius.

In astronomy a unit of solid angle is used called the *square degree* ($\square^\circ$)—a solid angle whose cone is a tetrahedral pyramid with an angle between its edges equal to 1°.

$1 \square^\circ = 3.046 \times 10^{-4}$ sr $= 2.424 \times 10^{-5}$ solid angle of a complete sphere.

The remark on the possibility of introducing an independent non-system unit of angle on a plane also relates to the measurement of solid angle, with the only difference that in the latter instance only the right angle is used in practice as such a unit (except for the astronomical unit—the square degree).

In connection with the above the following remark should be made. In the SI system, the units of angle and solid angle—the radian and steradian—have been placed into a separate group of "supplementary units". It seems to us that such a separation is absolutely unsubstantiated and may lead to misunderstanding, in particular it will make us consider these units to be outside of any system whatsoever. Actually, however, as noted above, the circumstance that the radian and steradian have no dimension with respect to the basic units of a system does not at all mean that they are non-system units. Equations (4.5) and (4.6) giving definitions of plane and solid angles are typical defining relationships in which the factor of proportionality, as usual, is assumed to equal unity and to be deprived of a dimension. Thus it should be considered that the units of plane and solid angles—the radian and steradian—are full-fledged derived units, with the only distinguishing feature that these units are the same in all systems.

If it is considered that dimensionless units should be placed in a special group of "supplementary units" instead of being included among the basic or derived ones, then this group should also include such quantities relating to the theory of oscillations (see below) as phase, quality, and, naturally, any dimensionless combinations of quantities, in particular the criteria of similarity mentioned above.

Curvature. Any curved line has a certain curvature at each point. An element of the curve adjoining the given point can be considered as part of a circle with a certain radius r (Fig. 7). The quantity that is a reciprocal of r,

$$\rho = \frac{1}{r} \qquad (4.7)$$

serves as a measure of curvature of a curve at a given point, while the radius r itself is called the radius of curvature. Thus, the unit of curvature is defined as the curvature at

such a point at which the radius of curvature of the given curve is equal to a unit of length.

Curvature of a surface. When we have to deal with a surface, the concept of curvature becomes more complicated. If at any point of a surface planes N_1 and N_2 are drawn perpendicular to a tangent (Fig. 8), then the intersection of the surface with these planes gives two curved lines A_1MB_1 and A_2MB_2, which may be characterized by the corresponding radii of curvature $r_1 = O_1M$ and $r_2 = O_2M$, and the curvature $\rho_1 = 1/r_1$ and $\rho_2 = 1/r_2$.

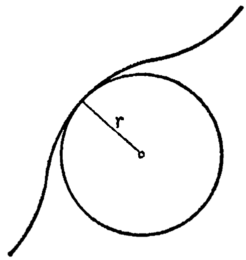

Fig. 7.

It is proved in differential geometry that no matter how these two intersecting planes are drawn, the sum

$$\rho = \rho_1 + \rho_2 = \frac{1}{r_1} + \frac{1}{r_2} \tag{4.8}$$

will remain constant. This sum is called the mean curvature at the given point of the surface. Sometimes half of

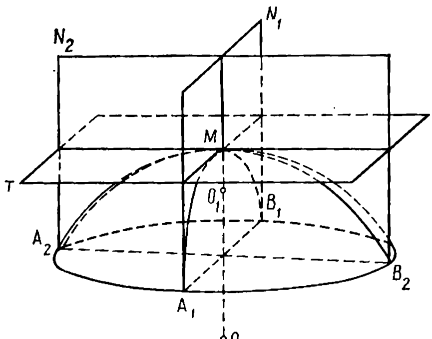

Fig. 8

this value is called the mean curvature:

$$\rho' = \frac{1}{2}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \tag{4.8a}$$

The mean curvature of a sphere will obviously be

$$\rho=\frac{2}{r} \tag{4.9}$$

or

$$\rho'=\frac{1}{r} \tag{4.9a}$$

where r is the radius of the sphere.

In addition to the mean curvature, a surface is sometimes characterized by the *Gaussian curvature*, determined by the expression

$$K=\frac{1}{r_1 r_2} \tag{4.10}$$

where r_1 and r_2 are the same as in formula (4.8).

For a sphere

$$K=\frac{1}{r^2} \tag{4.11}$$

The dimension both of the curvature of a curved line and of the mean curvature of a surface is

$$[\rho]=[\rho']=L^{-1} \tag{4.12}$$

The dimension of the Gaussian curvature is

$$[K]=L^{-2} \tag{4.13}$$

The unit of curvature of a curve in the SI and mk(force)s systems is the *inverse metre*—the curvature of a curve whose radius of curvature at the given point is equal to one metre. In the cgs system the unit of curvature is correspondingly the *inverse centimetre*. The units of mean curvature of a surface are also the inverse metre and inverse centimetres and ρ equals unity for a sphere with a radius of two metres [in the SI and mk(force)s systems], two centimetres (in the cgs system), or for a cylinder with a radius of one metre or one centimetre. Correspondingly ρ' equals unity for a sphere with a radius of one metre or one centimetre or for a cylinder with a radius of 0.5 metre or 0.5 centimetre. The units of Gaussian curvature (1/m^2) and (1/cm^2) are the Gaussian curvatures of spheres with radii of one metre or one centimetre. Obviously, $1\ \text{m}^{-1} = 10^{-2}\ \text{cm}^{-1}$ and $1\ \text{m}^{-2} = 10^{-4}\ \text{cm}^{-4}$.

Moments of plane figures. In strength of materials wide use is made of special geometrical characteristics of plane figures that are, for example, sections of various structural members. These characteristics also have the corresponding units.

The statical moment relative to an axis is a quantity determined as

$$S_z = \int_A r\,dA \tag{4.14}$$

where dA is an element of area, and r is the distance from this element to the axis relative to which the moment is being determined (Fig. 9). Integration is carried out over the entire area of the figure. The dimension of the statical moment is

$$[S_z] = L^3 \tag{4.15}$$

and its units are m^3 and cm^3.

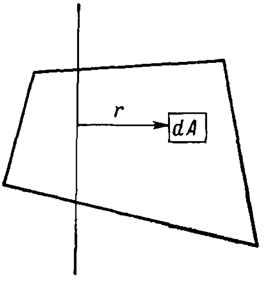

Fig. 9

The dimension and designations of the units of the statical moment coincide with those of the units of volume, although there is nothing in common between these quantities. This can serve as an illustrative example of the fact that the coincidence of dimensions does not at all mean the coincidence of the physical (or in the given case geometrical) essence of the quantities.

In accordance with formula (4.14), the statical moment of a rectangle with sides a and b relative to side b is

$$S_b = \frac{a^2 b}{2} \tag{4.16}$$

For this reason the statical moment of a rectangle with sides of 1 m and 2 m (or correspondingly 1 cm or 2 cm) relative to the side with a length of 2 m or, respectively, 2 cm may be taken as the unit of statical moment.

According to the dimension of the statical moment

$$1\ m^3 = 10^6\ cm^3$$

The *axial* (*equatorial*) *moment of inertia* is determined (see Fig. 9) as

$$J_z = \int_A r^2\, dA \tag{4.17}$$

The dimension of the axial moment of inertia is

$$[J_z] = L^4 \tag{4.18}$$

and its units are m^4 and cm^4:

$$1\ m^4 = 10^8\ cm^4$$

For a rectangle with sides a and b the axial moment of inertia relative to side b is

$$J_b = \frac{a^3 b}{3} \tag{4.18a}$$

and correspondingly the unit may be the moment of inertia of a rectangle with sides of one and three metres (centimetres) relative to the second of these sides.

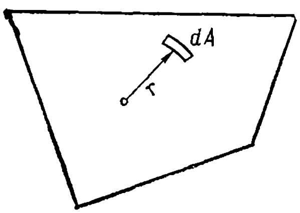

Fig. 10

The *polar moment of inertia* is computed in the same way as its axial counterpart, the only difference being that the distance is taken not to an axis, but to a certain definite point (Fig. 10):

$$I_0 = \int_A r^2\, dA \tag{4.19}$$

The dimension and units of the polar and axial moments of inertia coincide. From formula (4.19) it is easy to find the polar moment of inertia of a circle:

$$I_0 = \frac{\pi r^4}{2} \tag{4.20}$$

For this reason the polar moment of inertia of a circle whose radius is $\sqrt[4]{2/\pi} = 0.89$ metre or centimetre can be taken as the unit of the polar moment of inertia.

4.3. Kinematic Units

Time. Since kinematics considers processes of motion, a unit of time is required. In all the systems such a unit is the second, defined above as one of the basic units. Greater non-system units of time are the *minute*, 1 min = 60 s and the *hour*, 1 h = 60 min = 3 600 s. Fractions of the unit of time are constructed according to the decimal principle, i.e., *millisecond* (ms), *microsecond* (μs), *nanosecond* (ns).

Velocity. The unit of velocity is found from the formula for uniform rectilinear motion

$$v = \frac{l}{t} \tag{4.21}$$

According to this formula, the unit of velocity is the velocity of such uniform rectilinear motion at which a point travels a unit of length in a unit of time. The formula of uniform motion also determines the dimension of velocity

$$[v] = LT^{-1} \tag{4.22}$$

In the SI and mk(force)s systems the unit of velocity is the *metre per second* (m/s), and in the cgs system—the *centimetre per second* (cm/s).

It should be noted that 1 m/s is sometimes called a *mes*, though this name has not been legalized. Of the non-system units of velocity the most widely used in everyday life is the *kilometre per hour*, 1 km/h = 0.278 m/s.

In navigation the unit of velocity is the *knot*, equal to one nautical mile per hour or 1.852 km/h.

Acceleration. The unit of acceleration is established on the basis of the formula for uniformly accelerated motion

$$a = \frac{v_2 - v_1}{t} \tag{4.23}$$

where v_1 = initial velocity
v_2 = final velocity
t = time
a = acceleration.

Acceleration can be defined as the increase in velocity in a unit of time. Hence the unit of acceleration is taken as the acceleration of such uniformly accelerated rectilinear

motion at which the increase of velocity in a unit of time is equal to a unit of velocity. The dimension of acceleration is found from formula (4.23)

$$[a] = LT^{-2} \tag{4.24}$$

The unit of acceleration in the SI and mk(force)s systems is the *metre per second per second* (m/s^2)—the acceleration of such uniformly accelerated motion at which the velocity grows by 1 m/s every second. In the cgs system the unit of acceleration is correspondingly the *centimetre per second per second* (cm/s^2). This unit is sometimes (mainly in geophysics when measuring the acceleration due to gravity) called the *gal* (in honour of Galileo). The relationship between the units of acceleration is

$$1\ \text{m/s}^2 = 10^2\ \text{cm/s}^2$$

The unit of acceleration equal to the normal acceleration due to gravity, 9.81 cm/s^2, is widely used in aviation and astronautics. This unit is designated g. Acceleration measured in these units is often called overload, since it shows how many times the weight of a body moving with the given acceleration is greater than the weight of the same body at rest or moving uniformly near the surface of the Earth.

Angular velocity. In uniform rotation the angular velocity is equal to the ratio of the angle of rotation of a body to the time during which this rotation took place:

$$\omega = \frac{\varphi}{t} \tag{4.25}$$

The dimension of angular velocity is

$$[\omega] = T^{-1} \tag{4.26}$$

The unit of angular velocity is the angular velocity of uniform rotation at which a body turns through one radian in a unit of time (rad/s).

Angular acceleration. Angular acceleration is defined as the increase in the angular velocity in a unit of time

$$\alpha = \frac{\omega_2 - \omega_1}{t} \tag{4.27}$$

The unit of angular acceleration is the angular acceleration of such uniformly accelerated rotation at which the angular velocity increases in a unit of time by a unit of angular velocity (by one radian per second).

Since in all the systems the unit of time is the second, then the units of angular velocity and angular acceleration will be the same in all of them. The dimension of angular velocity is determined by formula (4.26), and of angular acceleration by the formula

$$[\alpha] = T^{-2} \tag{4.28}$$

For the non-system units of angle, the corresponding units of angular velocity and angular acceleration are respectively 1 rev/s, 1°/s, 1′/s, 1″/s and 1 rev/s², 1°/s², 1′/s², 1″/s². The relationships between these units are the same as between the relevant units of angle.

As an example we shall note that the minute hand of a timepiece moves with the angular velocity

$$0.1°/\mathrm{s} = 6'/\mathrm{s}$$

Period. Any periodical process consists of a number of cycles. By a cycle is meant a complete set of repeating values of a periodically changing quantity. The interval of time required to complete one cycle is called a period. The dimension of period is

$$[\tau] = T \tag{4.29}$$

and its unit is the *second* (s).

Frequency. The number of cycles completed in a unit of time is called the frequency (ν). Obviously,

$$\nu = \frac{1}{\tau} \tag{4.30}$$

The dimension of frequency is

$$[\nu] = T^{-1} \tag{4.31}$$

The unit of frequency in all the systems is the *hertz* (Hz), a frequency equal to one cycle per second. The term "cycle per second" (cps or c/s) is still encountered in technical literature instead of hertz. In other words, the hertz is the frequency of such a periodic process that repeats every

second. In radio engineering multiple units are used—*kilohertz* (kHz), *megahertz* (MHz), etc. A unit called the *fresnel* and equal to 10^{12} Hz was introduced about 1930, but it has never been popular.

In uniform rotation the following relationship can be established between frequency and angular velocity:

$$\omega = 2\pi\nu = \frac{2\pi}{\tau} \tag{4.32}$$

The concept of angular velocity is also very useful, however, when applied to other periodical processes (for example, to rectilinear oscillations). Here the angular velocity or, as it is also called, the angular frequency is determined directly from equation (4.32).

Phase. The instantaneous state of an oscillatory process is characterized by phase, which is the argument of the periodical function describing the process. For instance, in harmonic oscillatory motion the deviation x from the position of equilibrium is described by the equation

$$x = A \sin \Phi \tag{4.33}$$

Here A is the amplitude, i.e., the maximum deviation from the position of equilibrium, and Φ is the phase. With an angular frequency ω, at the moment t from the beginning of registration of the oscillations

$$\Phi = \omega t + \varphi \tag{4.34}$$

where φ is the initial phase, i.e., the phase at the initial moment. It is obvious that phase is a dimensionless quantity.

In electrical engineering, phase and phase difference are sometimes measured by the unit "electrical degree", which is the interval of time corresponding to 1/360th of the period of an alternating current. Since in the USSR the frequency of alternating current in all electrical mains is 50 Hz, then an interval of time equal to 55.6 μs corresponds to an "electrical degree". In countries where a frequency of 60 Hz is employed, the corresponding figure will be 46.3 μs.

Volumetric flow rate. The volume of a fluid flowing through a cross section in a unit of time is called the volumetric

flow rate (Q_v). Its dimension is

$$[Q_v] = L^3T^{-1} \tag{4.35}$$

and its units are m^3/s and cm^3/s.

Volumetric flow rate density (q_v) is the flow rate per unit of cross-sectional area:

$$q_v = \frac{Q_v}{A} \tag{4.36}$$

Its dimension

$$[q_v] = LT^{-1} \tag{4.37}$$

coincides with that of velocity. This should be expected, since the volumetric flow rate density is actually the linear velocity of a flow.

Velocity gradient. When a fluid flows in such a way that its different layers move with different velocities, a special quantity called the velocity gradient is introduced:

$$[\mathrm{grad}\, v] = \frac{dv}{dl} \tag{4.38}$$

which is the increment of velocity per unit of distance between the layers in the direction of the change in velocity. The dimension of the velocity gradient is

$$[\mathrm{grad}\, v] = \left[\frac{dv}{dl}\right] = T^{-1} \tag{4.39}$$

while its units m/(s·m) and cm/(s·cm), are the same in all the systems and can be written as s^{-1}.

The velocity gradient is also applicable to the curvilinear motion of a solid body. For example, for a uniformly rotating disk, the velocity of its points grows from the centre to the periphery. If the angular velocity of the disk is ω, then at a distance r from the centre the linear velocity v is equal to ωr. Hence

$$\mathrm{grad}\, v = \frac{dv}{dr} = \omega \tag{4.40}$$

Thus, the velocity gradient is here equal to the angular velocity of rotation of the disk and, consequently, is the same for all points of the disk.

4.4. Static and Dynamic Units

Mass. Previously we have already defined the basic units of mass in the SI system (*kilogram*) and the cgs system (*gram*), and also the technical unit of mass (or as we have agreed to call it in this book, the *inerta*), which is a derived unit in the mk(force)s system. We should only remember, however, that the latter has the dimension

$$[m] = L^{-1}FT^2 \tag{4.41}$$

and according to the relevant USSR State Standard is designated kgf·s²/m.

Among the units of mass obtained from the basic ones according to the decimal principle, the most widely used are the *ton*, equal to 10^3 kg, which was previously a basic unit in the metre-ton-second system, the *centner*, or *quintal* (q) equal to 100 kg, the *milligram* (mg) equal to 10^{-3} g, and the *microgram* (μg) equal to 10^{-6} g. In German and English technical literature, the name *gamma* (symbol γ) is sometimes used instead of microgram. The mass of precious stones is generally measured using a special unit named the *carat* equal to 2×10^{-4} kg or 0.2 g.

It is sometimes useful to have a unit of mass containing a definite number of molecules. Such a unit of mass will naturally differ for different substances. Such a unit is the *kilogram-molecule* or *kilomole*—the amount of a substance containing the same number of kilograms as there are units in the molecular weight of the substance. A unit 1 000 times smaller is called the *gram-molecule* or simply *mole*. A kilomole of any substance contains a constant number of molecules (Avogadro's number), equal, according to numerous investigations, to $(6.0249 \pm 0.0002) \times 10^{26}$; a mole correspondingly contains $(6.0249 \pm 0.0002) \times 10^{23}$ molecules. In rounded numbers a mole of hydrogen contains 2 grams, of oxygen 32 grams, of water 18 grams, etc.

Force. It should be remembered that the derived units of force in the SI and cgs systems, determined from Newton's second law, have the dimension

$$[f] = LMT^{-2} \tag{4.42}$$

and are respectively called the *newton* (N) and the *dyne* (dyn). The newton is defined as the force imparting to a mass

of 1 kg an acceleration of 1 m/s², and the dyne as the force imparting to a mass of 1 g an acceleration of 1 cm/s². It can therefore be written that

$$1\ \mathrm{N} = 1\ \mathrm{kg \cdot m/s^2}$$

$$1\ \mathrm{dyn} = 1\ \mathrm{g \cdot cm/s^2}$$

and

$$1\ \mathrm{N} = 10^5\ \mathrm{dyn}$$

The unit of force of the abolished metre-ton-second system is sometimes encountered in literature. This unit of force, the *sthene* (sn), is defined as the force imparting to a mass of 1 ton an acceleration of 1 m/s²;

$$1\ \mathrm{sn} = 1\ \mathrm{t \cdot m/s^2}$$

Obviously, 1 sn = 10^3 N.

The unit of force in the mk(force)s system—kgf*, which is the basic one in this system, is determined by a prototype

$$1\ \mathrm{kgf} = 9.81\ \mathrm{N} = 9.81 \times 10^5\ \mathrm{dyn}$$

The ton (force), equal to 10^3 kgf, and the gram (force), equal to 10^{-3} kgf, are also sometimes used in practice.

Impulse. The impulse of a force is measured by the product of the force and the duration of its action, *ft*. The unit of impulse is the impulse of a force equal to unity and acting during a unit of time. Accordingly, the units of impulse in the different systems are:

$$\mathrm{SI}: 1\ \mathrm{N \cdot s} = 1\ \mathrm{kg \cdot m/s}$$

$$\mathrm{cgs}: 1\ \mathrm{dyn \cdot s} = 1\ \mathrm{g \cdot cm/s}$$

$$\mathrm{mk(force)s}: 1\ \mathrm{kgf \cdot s}$$

Momentum (*impulse*). Momentum is defined as the product of the mass of a body and its velocity (*mv*). The unit of momentum of a body is the momentum of a body with a unit mass moving with a unit velocity.

We have previously mentioned (see Sec. 2.1.) that in modern literature on physics the term "impulse of a body"

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* According to the standard definition, a kilogram (force)—kgf—is the force that imparts to a mass of 1 kg the normal acceleration of gravity, i.e., 9.80665 m/s².

is often used instead of the term "momentum". From the formula

$$ft = mv_2 - mv_1 \tag{4.43}$$

(the impulse is equal to the change in the momentum) it follows that the dimension of impulse and momentum should coincide. Indeed,

$$[ft] = LMT^{-2}T = LMT^{-1} \tag{4.44}$$

$$[mv] = MLT^{-1} = LMT^{-1} \tag{4.45}$$

The units of momentum and of impulse are the same, in the SI system—kg·m/s, in the cgs system—g·cm/s; in the mk(force)s system the unit can be written as i·m/s, which, of course, is equal to kgf·s

$$1 \text{ kg}\cdot\text{m/s} = 10^5 \text{ g}\cdot\text{cm/s}$$

$$1 \text{ kgf}\cdot\text{s} = 9.81 \text{ kg}\cdot\text{m/s}$$

Pressure. With a uniformly distributed load, pressure is determined by the force acting per unit of area,

$$p = \frac{f}{A} \tag{4.46}$$

The unit of pressure is such a uniformly distributed pressure when a unit of force acts on a unit of surface. The dimension of pressure is

$$[p] = L^{-1}MT^{-2} \tag{4.47}$$

In the SI system the unit of pressure is the *newton per square metre* (N/m^2). It has been proposed to call this unit the *pascal* (Pa). In the cgs system the unit is *dyne per square centimetre* (dyn/cm^2). The dimension of pressure establishes the relationship between the units N/m^2 and dyn/cm^2:

$$1 \text{ N/m}^2 = 10 \text{ dyn/cm}^2$$

In the mk(force)s system the unit of pressure is kgf/m^2, which is equal, obviously, to 9.81 N/m^2 = 98.1 dyn/cm^2. A pressure of 1 kgf/m^2 is, with a high degree of accuracy, equal to the pressure of a column of water 1 mm high. Indeed, a layer of water with an area of 1 m^2 and a thickness of 1 mm occupies a volume equal to 1 dm^3, consequently its weight with a high accuracy is equal to 1 kgf. For this reason in engineering the unit of pressure kgf/m^2 is often

called a millimetre of water column (mm of water, or mm H_2O). This is especially convenient when water pressure gauges are used (for example, when measuring the velocity of a gas in a pipe).

The unit of pressure in the cgs system dyn/cm^2 in literature on physics was previously called the *bar*. In meteorology this name is used to denote a unit 10^6 times greater, and equal to 10^5 N/m^2. The bar is given this value in GOST 7664-55 covering non-system mechanical units. It is not recommended to use this name to denote dyn/cm^2.

In addition to the units mentioned above, a number of non-system units are widely used in physics and engineering. One of them that is in great favour is the *standard atmosphere*—the pressure of air balanced by a column of mercury 76 cm high with a density of the mercury of 13.595 g/cm^3 at normal acceleration of gravity. Such a column applies a pressure equal to its weight to each square centimetre. The exact value of a standard atmosphere is

$$1 \text{ atm} = 76 \text{ cm} \times 13.595 \text{ g/cm}^3 \times 980.665 \text{ cm/s}^2 = \\ = 1.01325 \times 10^6 \text{ dyn/cm}^2 = 1.01325 \times 10^5 \text{ N/m}^2$$

Since this pressure is approximately equal to 1.033 kgf/cm^2, instead of it use is often made of the *technical atmosphere* (at), exactly equal to 1 kgf/cm^2. Obviously, 1 at $=$ $= 10^4$ kgf/m^2 $= 9.81 \times 10^4$ N/m^2. In the metre-ton-second system the unit of pressure was called the *pieza* (pz) and was equal to a pressure of 1 sthene per square metre. Hence

$$1 \; pz = 10^3 \text{ N/m}^2 = 10^4 \text{ dyn/cm}^2 = 0.01 \text{ bar}$$

(here a bar is equal to 10^6 dyn/cm^2).

The pressure is quite frequently measured directly in millimetres of mercury column (mm Hg), also called *torrs* (after Torricelli). The latter name is seldom encountered in literature on the subject.

Obviously, 1 mm Hg (1 torr) $= 10^{-3}$ m $\times$ 13.595 $\times$ 10^3 kg/m^3 $\times$ 9.80665 m/s^2 $=$ 133.3 N/m^2 $=$ 1 333 dyn/cm^2.

The system units of pressure and their multiples and submultiples are also used to measure mechanical stresses.

Pressure gradient. The flow of fluids in channels and pipes is determined by the pressure difference per unit of length

of the stream. For a constant cross section of the stream this quantity can be written as

$$\frac{p_1 - p_2}{l_2 - l_1} \tag{4.48}$$

where l is the distance along the stream measured from its origin. When the cross section of the stream is not uniform, the expression $-\frac{dp}{dl}$ should be substituted for expression (4.48).

The quantity

$$\frac{dp}{dl} = \text{grad}\, p \tag{4.49}$$

is called the pressure gradient.

It is easy to see that the dimension of the pressure gradient is

$$[\text{grad}\, p] = L^{-2}MT^{-2} \tag{4.50}$$

where grad p is measured in units of pressure related to a unit of length

$$\frac{\text{N}}{\text{m}^2\cdot\text{m}} = \frac{\text{N}}{\text{m}^3} = \frac{\text{kg}}{\text{m}^2\cdot\text{s}^2}; \quad \frac{\text{dyn}}{\text{cm}^2\cdot\text{cm}} = \frac{\text{dyn}}{\text{cm}^3} = \frac{\text{g}}{\text{cm}^2\cdot\text{s}^2}; \quad \frac{\text{kgf}}{\text{m}^2\cdot\text{m}} = \frac{\text{kgf}}{\text{m}^3}$$

and in the non-system units

$$\frac{\text{atm}}{\text{m}}; \quad \frac{\text{torr}}{\text{cm}}; \quad \text{etc.}$$

The dimension of the pressure gradient determines the relationship between the units:

$$1\ \text{N/m}^3 = 0.1\ \text{dyn/cm}^3; \quad 1\ \text{kgf/m}^3 = 9.81\ \text{N/m}^3$$

Work and energy. Work under the action of a constant force is defined as the product of the force, the distance, and the cosine of the angle between their directions

$$W = fl\cos(\widehat{f, l}) \tag{4.51}$$

The unit of work is the work done by a unit of force over a path equal to a unit of length when the force and the path coincide in direction. The dimension of work is determined from its formula

$$[W] = [f]\,[l] = L^2MT^{-2} \tag{4.52}$$

In the SI system the unit of work is the *joule* (J) — the work performed by one newton over a distance of 1 metre:

$$1\ \mathrm{J} = 1\ \mathrm{N \cdot m}$$

In the cgs system the unit of work — the *erg* — is the work performed by one dyne over a distance of 1 cm:

$$1\ \mathrm{erg} = 1\ \mathrm{dyn \cdot cm}$$

In the mk(force)s system the unit of work—the *kilogram-metre*—is the work done by 1 kilogram force over a distance of 1 metre (1 kgf·m). The relationship between these units of work is obtained from formula (4.52)

$$1\ \mathrm{kgf \cdot m} = 9.81\ \mathrm{J};\quad 1\ \mathrm{J} = 10^7\ \mathrm{erg}$$

Before the introduction of the International System of Units, special heat units of work—units of the quantity of heat—the *calorie* (cal) and *kilocalorie* (kcal) were used in all thermal calculations. The first of these units can be approximately defined as the quantity of heat required to heat 1 gram of water by 1°C; 1 kcal = 1 000 cal. These units will be considered in greater detail together with the other thermal units. In connection with the introduction of the International System of Units, it is recommended to use the general units of work—the joule and its multiples and submultiples—instead of the calorie and kilocalorie. The following relationship has been established for the conversion of calories to joules:

$$1\ \mathrm{cal} = 4.1868\ \mathrm{J}$$

(the coefficient 4.1868 is sometimes called the mechanical equivalent of heat).

When measuring the work performed in various processes involving a change in the state of a gas, use is sometimes made of a unit that is determined according to the work of expansion of the gas at constant pressure

$$W = p\Delta V \tag{4.53}$$

where p is the pressure of the gas and ΔV the change in its volume. If $p = 1$ atm and $\Delta V = 1$l, then the corresponding work can be called a *litre-atmosphere* (l·atm):

$$1\ \mathrm{l \cdot atm} = 10^{-3}\ \mathrm{m^3} \times 1.01325 \times 10^5\ \mathrm{N/m^2} = 1.01325 \times 10^2\ \mathrm{J}$$

Both potential energy E_p, also called the energy of rest, and kinetic energy E_k, or the energy of motion, are measured in the same units as work. The sum of both kinds of energy gives the total energy of a system:

$$E = E_k + E_p \tag{4.54}$$

Energy density. Sometimes energy is stored in a certain volume. For instance, a compressed gas has a certain stock of energy uniformly distributed through its volume. The energy per unit of volume is called the energy density e:

$$e = \frac{E}{V} \tag{4.55}$$

Its dimension is

$$[e] = L^{-1}MT^{-2} \tag{4.56}$$

and its units are J/m^3, erg/cm^3, $kgf \cdot m/m^3$, and kgf/m^2.

Since the dimension of energy density coincides with that of pressure, then the relationship between the units will also be the same:

$$1\ kgf/m^2 = 9.81\ J/m^2;\ 1\ J/m^3 = 10\ erg/cm^3$$

Power is the rate of performing work. The power of a uniformly working system, for example, a machine, mechanism or the like, is equal to the work performed in a unit of time

$$P = \frac{W}{t} \tag{4.57}$$

The unit of power is the power of such a uniformly working system that performs a unit of work in a unit of time. The dimension of power is

$$[P] = L^2MT^{-3} \tag{4.58}$$

The unit of power in the SI system is the *watt*, or *joule per second* (W = J/s), in the cgs system the *erg per second*, and in the mk(force)s system the *kilogram-metre per second* (kgf·m/s). Since the unit of time in all these systems is the second, then the relationship between the units of power remains the same as between those of work.

Larger and smaller decimal multiple and submultiple units of power, the *kilowatt* (kW), *megawatt* (MW), *milliwatt* (mW) and *microwatt* (μW), are in great favour. The *hectowatt* (hW) is used less frequently. The watt and its decimal-

derivatives are almost exclusively used to form units of energy for measuring electrical energy. These units are the *watt-hour* (W-h), *hectowatt-hour* (hW-h), *kilowatt-hour* (kW-h), and *megawatt-hour* (MW-h). They denote the work done at the corresponding power during one hour. It follows from this definition that 1 W-h = 3 600 J, 1 hW-h = $= 3.6 \times 10^5$ J = 360 kJ, 1 kW-h = 3.6×10^6 J = 3.6 MJ, and 1 MW-h = 3 600 MJ.

The thermal units of work related to a unit of time give the corresponding units of power. Of these units the ones in greatest use are the *calorie per hour* (cal/h) and the *kilocalorie per hour* (kcal/h). The relationship between the kilocalorie per hour and the watt is

$$1 \text{ kcal/h} = 1.163 \ W$$

Up to the present time quite widespread use is made of a unit of power with the absurd name *horsepower* (hp), both in the metric system, and in the system of weights and measures still in use in some English-speaking countries. These "horsepowers" differ slightly in value. The metric horsepower is equal to 75 kgf·m/s or 736 W, while the British (or U.S.) horsepower is equal to 550 ft-lb/s or 745.7 W.

Coefficient of friction. When a body moves along a surface, a braking force appears—the force of friction. Its magnitude f_{fr} depends on the nature of the contacting surfaces and is proportional to the normal force f_n of the pressure forcing the body against the surface:

$$f_{fr} = C_{fr} f_n \tag{4.59}$$

The coefficient C_{fr} is called the coefficient of friction and, according to its definition, is a dimensionless quantity, the same in all systems.

Coefficient of resistance. If a body moves in a viscous medium (a gas or liquid), a force of resistance appears that depends on the velocity. At relatively low velocities this force is proportional to the velocity:

$$f = rv \tag{4.60}$$

The coefficient r, called the coefficient of resistance, depends on the properties of the medium, and the dimensions and

shape of the body. Its dimension is

$$[r] = MT^{-1} \tag{4.61}$$

and its units are N·s/m = kg/s, dyn·s/cm = g/s, and kgf·s/m. The relationships between these units are the same as between the units of mass, namely, 1 N·s/m = $= 10^3$ dyn·s/cm, 1 kgf·s/m = 9.81 m·s/m.

Flexibility. If an external force is applied to an elastic system, the latter is deformed. If Hooke's law is observed, then the linear deformation is proportional to the applied force

$$\Delta x = kf \tag{4.62}$$

The coefficient k is called the flexibility of the system. Formula (4.62) determines the dimension of flexibility:

$$[k] = M^{-1}T^2 \tag{4.63}$$

and its units m/N, cm/dyn and m/kgf.

According to equation (4.63), 1 m/N = 10^{-3} cm/dyn, 1 m/kgf = 1/9.81 m/N = 0.102 m/N.

Moment of force. The moment of a force relative to a certain point is measured by the product of the force and its arm, i.e., the distance between the direction of the force and the point relative to which the moment of the force is taken. For this reason the unit of moment of a force can be taken equal to the moment of a force equal to unity with an arm equal to a unit of length. From the formula for the moment of a force

$$M = fl \tag{4.64}$$

where M is the moment of the force f and l is the arm, it follows that the dimension of the moment of a force is

$$[M] = L^2MT^{-2} \tag{4.65}$$

We see that the dimension of the moment of a force coincides with that of work and energy. It should be noted, however, that these quantities are of an absolutely different nature. While work and energy do not have a direction, and are scalar quantities, the moment of a force has a direction, i.e., is a vector quantity.

The units of moment of a force are formed in the same way as the units of work, but the names erg, joule, etc. are

not used with respect to it. Thus, for moment of a force we have the units N·m, dyn·cm, and kgf·m.

Moment of inertia of a body (*dynamic moment*). In mechanics, especially when considering the rotary motion of a body, it is very convenient to use a special quantity—the moment of inertia of a body—which is calculated relative to a certain axis.

For purposes of illustration let us first find the moment of inertia of a material point relative to an axis. It is equal to

$$I = mr^2 \tag{4.66}$$

where m is the mass of the material point, and r the distance from it to the axis relative to which the moment of inertia is being determined.

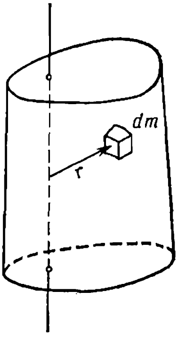

Fig. 11

For a system of rigidly connected material points or for a solid body the moment of inertia can be determined as the sum of the products of the masses of the separate material points which the system consists of or which system can be divided into and the squares of the corresponding radii—the distances to the axis of rotation (Fig. 11):

$$\text{for a system of points } I = \Sigma mr^2 \tag{4.67}$$

$$\text{for a solid body } I = \int\limits_V r^2\,dm \tag{4.68}$$

In the process of studying problems connected with the rotation of bodies, the essence and role of the moment of inertia are ascertained. It is found that all the formulas describing the rotary motion of a solid body have a form similar to the corresponding formulas for translational motion if the linear quantities in the latter (velocity, acceleration) are replaced by the relevant angular quantities (angular velocity, angular acceleration), and the mass by the moment of inertia relative to the axis of rotation. The unit of moment of inertia follows from its definition, and is the moment of inertia of a material point having a mass equal to unity, with a distance to the axis equal to a unit of length. The dimension of moment of

inertia is accordingly

$$[I] = L^2M \tag{4.69}$$

and its units are kg·m², g·cm², and i·m² (1 i·m² = = 1 kgf·m·s²). The relationships between them are 1 kg·m² = = 10^7 g·cm², and 1 kgf·m·s² = 9.81 kg·m².

Impulse of moment of force. The impulse of the moment of a force relative to a certain axis is the product of the moment of the force relative to this axis and the duration of action of the force

$$Mt = flt \tag{4.70}$$

The formula defining the impulse of the moment of a force gives for its dimension

$$[Mt] = L^2MT^{-1} \tag{4.71}$$

Moment of momentum. The moment of momentum (also called angular momentum) of a material point rotating about an axis is the product of the momentum of this point and the distance to the axis of rotation.

$$\mathcal{L} = \bar{p}r = mvr \tag{4.72}$$

Since the linear velocity in rotation can be expressed through the angular velocity by the formula

$$v = \omega r$$

then the moment of momentum can be written as

$$\mathcal{L} = I\omega \tag{4.73}$$

Thus, the moment of momentum is equal to the product of the moment of inertia of a rotating point relative to the axis of its rotation and the angular velocity. It follows from formulas (4.72) and (4.73) that the dimension of the moment of momentum, similar to that of the impulse of the moment of a force, is

$$[\mathcal{L}] = L^2MT^{-1} \tag{4.74}$$

The equality of the dimensions of the impulse of the moment of a force and the moment of momentum, naturally, follows from the law that "the impulse of the moment of a force relative to the axis of rotation is equal to the change in

the moment of momentum"

$$Mt = \mathcal{L}_2 - \mathcal{L}_1 \tag{4.75}$$

The units of the impulse of the moment of a force and the moment of momentum also coincide. They can be defined as the impulse of the moment of a force equal to unity during a unit of time, or as the moment of momentum of a body having a moment of inertia (dynamic) equal to unity, and rotating with an angular velocity equal to unity. These units are:

$$\mathrm{N \cdot m \cdot s = kg \cdot m \cdot m/s = kg \cdot m^2/s}$$

$$\mathrm{dyn \cdot cm \cdot s = g \cdot cm \cdot cm/s = g . cm^2/s}$$

$$\mathrm{kgf \cdot m \cdot s = i \cdot m \cdot m/s = i \cdot m^2/s}$$

The relationships between them are the same as between the corresponding units of work or those of the moment of inertia, since the unit of time in all the systems is the same.

Action. A quantity called action and whose dimension is the product of energy and time plays an appreciable part in analytical and quantum mechanics and in a number of other branches of physics. Without going into its physical essence, it should be noted that its dimension coincides with that of the moment of momentum or the impulse of the moment of a force and is accordingly measured in the same units.

Mass flow. In investigating the flow of liquids and gases, in addition to the volumetric flow rate considered above, use is made of a quantity called the mass flow (Q_m). Its dimension is

$$[Q_m] = MT^{-1} \tag{4.76}$$

and its units are kg/s, g/s, and i/s (kgf·s/m). The relationships between them are obviously the same as between the units of mass.

Mass flow velocity. The mass flow velocity or mass flow density q_m is determined similar to the volumetric flow rate density as the mass flow related to a unit of cross-sectional area of the flow. The dimension of q_m is accordingly

$$[q_m] = L^{-2}MT^{-1} \tag{4.77}$$

and its units are kg/(s·m²), g/(s·cm²) and i/(s·m²).

Dynamic characteristics of oscillatory motion. Together with the kinematic quantities—frequency, period, phase and amplitude—an oscillating system is characterized by a number of dynamic quantities, including kinetic and

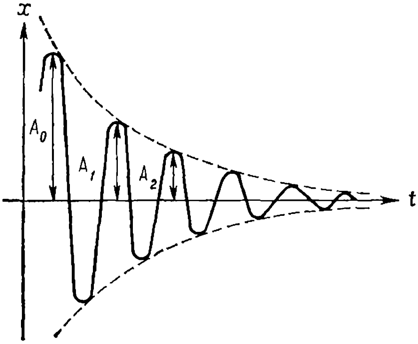

Fig. 12

potential energy and their units, considered above. Of great significance are the quantities characterizing the properties of an actual oscillating system. Since every such system has attenuation, or damping, its motion in the absence of an external force (Fig. 12) can be written as

$$x = A_0 e^{-\delta t} \sin(\omega t + \varphi) \tag{4.78}$$

where x = deviation from the position of equilibrium
A_0 = initial amplitude
e = base of natural logarithms
ω = angular velocity
φ = initial phase
δ = damping factor.

The dimension of δ

$$[\delta] = T^{-1} \tag{4.79}$$

determines its unit s^{-1}. Attenuation is also usually characterized by the logarithmic damping decrement $\varkappa$, equal to

$$\varkappa = \delta\tau \tag{4.80}$$

where τ is the period of oscillations. It can be seen from definition (4.80) that $\varkappa$ is a dimensionless quantity. It

can be easily shown from formulas (4.78) and (4.80) that the logarithmic decrement is equal to the natural logarithm of the ratio of two consecutive amplitudes. Another important characteristic of an oscillating system is its quality Q, determined by the formula

$$Q = 2\pi \frac{E_{tot}}{E_l} \tag{4.81}$$

where E_{tot} = total energy of the system in resonance
E_l = loss of energy during one period.

It is obvious that Q is a dimensionless quantity. It can be shown that

$$Q = \frac{1}{2\delta \sqrt{km}} \tag{4.82}$$

where k is the flexibility of the system [see formula (4.62)].

4.5. Units of Mechanical and Molecular Properties of a Substance

Both in scientific research and in numerous spheres of everyday life we utilize the most diverse properties of the materials that we have to deal with. These properties, as a rule, are characterized by definite quantities and can be measured quantitatively in some way or other. Different materials have different mechanical strength, different elasticity, etc.

In addition, any substance also has a number of molecular characteristics, connected either with its structure or with the processes in which its molecular nature appears. In this section we shall mainly consider the units of the mechanical and molecular properties of a substance which we have to deal with in physics and related subjects.

Density. Density is defined as the ratio of the mass of a homogeneous body to its volume

$$\rho = \frac{m}{V} \tag{4.83}$$

The unit of density is the density of such a homogeneous substance whose amount in a unit of volume is equal to the unit of mass. The unit of density in the SI system is kg/m^3, in the cgs system g/cm^3, and in the mk(force)s system i/m^3

or kgf·s²/m⁴. Frequently, especially when dealing with a gas, the density is measured in g/l. The value of the density measured in g/l coincides with that expressed in kg/m³. The dimension of density

$$[\rho] = L^{-3}M \tag{4.84}$$

allows the relationship between its various units to be established quite easily *.

Specific volume. The reciprocal quantity of density is called the specific volume

$$v = \frac{1}{\rho} = \frac{V}{m} \tag{4.85}$$

All the units of specific volume are reciprocals of the corresponding units of density. In the same way its dimension is

$$[v] = L^3M^{-1} \tag{4.85a}$$

Specific weight is the ratio of the weight of a body to its volume

$$\gamma = \frac{f}{V}$$

Since the weight of a body and its mass are related by the formula

$$f = mg \tag{4.86}$$

where g is the acceleration of gravity, then the relationship between the density ρ and the specific weight γ will be

$$\gamma = \rho g \tag{4.87}$$

The unit of specific weight is the specific weight of such a homogeneous substance, a unit of volume of which is attracted to the Earth with a force equal to a unit of force. Thus we obtain the units N/m³, dyn/cm³ and kgf/m³. For example, the specific weight of water in the cgs system is 981 dyn/cm³. Specific weight is often measured in gf/cm³ and kgf/*l*. Both these values coincide and are practically equal to the specific weight of water at 4°C. The numerical

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* When measuring the density of a liquid with an areometer various conditional scales were previously used (see Appendix 2).

value of specific weight expressed in gf/cm^3 or kgf/l coincides with that of density expressed in g/cm^3 or in kg/l.

Molecular weight (*molecular mass*, *relative molecular mass*) is the ratio of the mass of a molecule of a given substance to the so-called atomic unit of mass. Previously, in chemistry, this unit was taken equal to one-sixteenth of the mass of an atom of oxygen (the mean value of the mass of the three isotopes of oxygen with account taken of their relative contents in per cent). In 1961 the atomic unit of mass was taken equal to one-twelfth of the mass of the isotope of carbon with a mass number of 12 (i.e., containing six protons and six neutrons in its nucleus). This unit will be considered in greater detail in Sec. 9.2.

As mentioned above (Sec. 4.4), the mass of a substance containing the same number of kilograms or grams as there are units in the molecular weight of the given substance is called the kilomole or mole.

Molecular volume is the volume occupied by one kilomole or mole of a substance. Molecular volume in standard conditions, i.e., at 0°C and a pressure of 1 atm, is called standard molecular volume. It is equal to 22.42 m^3 for a kilomole and 22.420 cm^3 (22.42 l) for a mole.

The *number of kilomoles* (or *moles*) contained in a given mass of a substance can be determined as the ratio of this mass to that of one kilomole or mole. If we define the latter as the number of kilograms per kilomole or grams per mole and designate it in the same way as molecular weight, we can find the number of kilomoles or moles from the expression

$$Z = \frac{m}{M} \tag{4.88}$$

Coefficients of extension and shear and moduli of elasticity and shear. If a solid specimen is subjected to linear (uniaxial) tension or compression, it deforms (becomes stretched or compressed), its deformation or strain within certain limits following Hooke's law

$$\Delta l = \alpha \frac{lf}{A} \tag{4.89}$$

where Δl = deformation or strain
l = initial length

f = load
A = cross-sectional area.

The coefficient α is called the coefficient of extension of a material. It is the deformation of a specimen of unit length and unit cross-sectional area when a unit deforming force is applied to it. The dimension of this coefficient is

$$[\alpha] = LM^{-1}T^{2} \qquad (4.90)$$

In the SI system its unit is m^2/N, in the cgs system cm^2/dyn, and in the mk(force)s system m^2/kgf. In strength of materials the reciprocal quantity is generally used:

$$E = \frac{1}{\alpha} \qquad (4.91)$$

which is called the modulus of elasticity, or Young's modulus. Its units are reciprocals of those of the coefficient of extension—in the SI system N/m^2, in the cgs system dyn/cm^2, and in the mk(force)s system kgf/m^2. In engineering Young's modulus is often measured in kgf/cm^2 or kgf/mm^2. It is the load that would have to be applied to a specimen with a cross-sectional area equal to 1 m^2, 1 cm^2 or 1 mm^2 to double its length (if Hooke's law would be constantly observed during the process and the specimen did not fail).

Similar to the coefficient of extension and Young's modulus, there can be defined the *coefficient* and *modulus of shear*. The relationships between the units of Young's modulus or the modulus of shear are the same as between the units of pressure.

Coefficient of bulk compression. If a specimen is subjected to uniform or bulk (triaxial) compression under a certain pressure p, its volume will decrease by ΔV, determined from the formula

$$\Delta V = kpV \qquad (4.92)$$

where k is the coefficient of bulk compression. The unit and dimension of this coefficient obviously coincide with those of the coefficient of extension. In contrast to the latter, the concept of the coefficient of bulk compression is applicable not only to solid bodies, but also to fluids (liquids and gases). In this case it is usually called the compressibility factor. With a view to the greater compressibility

of gases, it is more convenient to write the compressibility factor as

$$k = -\frac{1}{V}\frac{dV}{dp} \tag{4.93}$$

The minus sign shows that a growth in pressure corresponds to a reduction in volume.

Hardness. The resistance of bodies to destruction or the formation of permanent set when sufficiently great deforming forces act on their surface is characterized by hardness. Since with a different nature of the action on the surface of a body it behaves in different ways, it is difficult to indicate a sufficiently objective and single-valued characteristic of hardness. Upon destruction of a solid body an attempt can be made to appraise hardness by the work of destruction related to a unit of area of the newly formed surface, taking into consideration the fact that the surface of the body increases upon destruction. With such a definition, hardness should be measured by the same units as the coefficient of surface tension (see below), determined by the free energy per unit of surface. It should be noted, however, that the actual work of destruction is considerably greater than the increase in the free energy of the surface, since the predominant part of the work done is dissipated as heat. The fact is also of importance that with different methods of treatment the work actually done may vary quite considerably. This is why different methods of conditionally appraising the hardness of materials have come into favour in engineering.

In mineralogy hardness scales are used in which numbers in growing order designate materials so arranged that each following one is capable of leaving a scratch on the surface of the previous one. Talc and diamond are at the extremes of these scales. The arrangement of minerals in the Mohs and Breithaupt scales is given in Table 47.

Methods of determining hardness are used in engineering that are based on measuring the dimensions of the indentations obtained when steel balls, diamond cones or prisms are pressed into the surface of the material being tested (Brinell, Rockwell or Vickers hardness). For purposes of illustration let us consider the method used to determine the Brinell hardness of a material. For this purpose a har-

dened steel ball is pressed into the surface of the material by a definite load, and the diameter d of the indentation formed is measured. If the diameter of the ball is D, and the load P, then the measure of hardness is the quantity BHN, determined by the formula

$$BHN = \frac{2P}{\pi D\left(D - \sqrt{D^2 - d^2}\right)} \tag{4.94}$$

Here P is measured in kgf, D and d in mm. Hence BHN is measured in kgf/mm^2.

Impact strength (ductility). In addition to hardness, the resistance of a material to destruction is characterized by the impact strength, also called ductility, which is measured by the work done for impact destruction of a specimen related to a unit of its cross-sectional area at the place of fracture. The units of impact strength are J/m^2, erg/cm^2 and kgf·m/m^2. The relationships between them are 1 J/m^2 = 10^3 erg/cm^2, and 1 kgf·m/m^2 = 9.8 J/m^2.

Viscosity (coefficient of internal friction). If laminary (striated) flow of separate layers takes place in a fluid, then a force directed tangentially to the surface of these layers appears in them. The existence of viscosity also leads to the appearance of a force acting on every body moving in a fluid or around which a fluid flows. This force, known as the force of internal friction, is expressed by the formula

$$f = -\eta \frac{dv}{dl} A \tag{4.95}$$

where dv/dl = velocity gradient
A = area acted upon by the force f
η = viscosity.

The minus sign in the formula shows that the force is directed toward the layer moving with a higher velocity.

Both the definition and the dimension of the unit of viscosity ensue from formula (4.95). Viscosity is measured by the force acting on a unit of surface area of one of the interacting layers from the other layer if the distance between the layers is equal to a unit of length and the layers move with respect to each other with a unit of velocity. The dimension of viscosity is

$$[\eta] = L^{-1}MT^{-1} \tag{4.96}$$

In the SI system the unit of viscosity has no special name and is designated N·s/m². In the cgs system its unit is the *poise* (P), defined as the viscosity of such a fluid in which at a velocity gradient of 1 cm/s per cm, a force of friction equal to one dyne acts on each square centimetre of a flowing layer. From the dimension formula of viscosity or its designation in the SI system it is easily found that 1 N·s/m² = 10 P. It can similarly be shown that the unit of viscosity in the mk(force)s system is 9.81 N·s/m² or 98.1 P.

The viscosity of water at 20.5°C is quite accurately equal to 0.01 P, i.e., 1 centipoise (cP). The viscosity of a fluid grows with a reduction in temperature. In particular, the viscosity of water at 0°C is 1.79 cP. The ratio of the viscosity of a fluid to that of water at the same temperature is called the relative viscosity. The ratio of the viscosity of a fluid to that of water at 0°C is called the specific viscosity. Both the relative and the specific viscosities are dimensionless quantities.

A great diversity of various instruments generally known as viscosimeters have been proposed for practical determination of viscosity. Some of them allow the viscosity to be determined in any of the units given above. Viscosimeters are also used, however, in which the value of viscosity is given in conditional units. Among the latter considerable use is made in the USSR (especially for measuring the viscosity of petroleum products) of the Engler viscosimeters, in which the duration of outflow of 200 grams of liquid is directly measured. This time serves as a measure of viscosity in so-called *Engler seconds*. The ratio of this time to the time of outflow of the same volume of water at a temperature of +20°C gives the viscosity of the liquid in *Engler degrees* (°E). The relationship between Engler degrees and poises is given by the approximate formula

$$\eta = \left(0.0731^{\circ}E - \frac{0.0631}{^{\circ}E}\right)\rho \qquad (4.97)$$

where η = viscosity in poises
°E = viscosity in Engler degrees
ρ = density of liquid in g/cm³.

Fluidity. A quantity that is the reciprocal of viscosity is called fluidity:

$$\varphi = \frac{1}{\eta} \tag{4.98}$$

The dimension of fluidity is

$$[\varphi] = LM^{-1}T \tag{4.99}$$

The unit of fluidity in the SI system is measured in $m^2/N \cdot s$, and in the cgs system—in reciprocal poises. Sometimes this unit is called and designated *rhe.*

Kinematic viscosity. Besides the viscosity considered above, which is frequently called the dynamic viscosity, wide use is made in hydrodynamics of kinematic viscosity, defined as the ratio of the dynamic viscosity to the density of a fluid:

$$\nu = \frac{\eta}{\rho} \tag{4.100}$$

The dimension of kinematic viscosity

$$[\nu] = L^2T^{-1} \tag{4.101}$$

coincides with that of the diffusion coefficient (see below). In accordance with its dimension, the unit of viscosity in the SI system is designated m^2/s. The unit is the same in the mk(force)s system. The unit of viscosity in the cgs system, cm^2/s, equal to 10^{-4} m^2/s, is called the *stokes* (st).

The *coefficient of surface tension* of a liquid is determined by the force acting on each unit of length of the boundary of the liquid film. This coefficient can also be defined as the free energy* of a unit of surface area of the liquid film. The dimension formula of the coefficient of surface tension follows from both these definitions:

$$[\sigma] = \frac{[f]}{[l]} = \frac{[E]}{[A]} = MT^{-2} \tag{4.102}$$

The unit of the coefficient of surface tension is defined as the coefficient of surface tension of such a liquid film on each unit of length of whose boundary there acts a unit

* The free energy is approximately defined as that part of the energy of a system that can be converted into work. A stricter definition is given in the course of thermodynamics.

force, or each unit of whose surface area has a free energy equal to unity.

The definition of the coefficient of surface tension and its unit as the free energy of a unit of surface area makes it possible to extend the concept of the coefficient of surface tension to solid bodies, since the molecules in the surface layer of a body have a higher potential energy than those inside it.

The unit of the coefficient of surface tension in the SI system is N/m or J/m^2, in the cgs system dyn/cm or erg/cm^2, and in the mk(force)s system kgf/m or kgf·m/m^2. Tables sometimes give this coefficient in milligrams force per millimetre (mgf/mm). It is simple to find that 1 mgf/mm = = 9.81 dyn/cm.

The dimension "particle". In molecular and atomic physics, together with macroscopic quantities (density, viscosity, etc.) we also have to deal with quantities characterizing the properties of separate particles—molecules, atoms, electrons, ions, etc. Such quantities as the energy of molecules, the mass of an atom and the charge of an electron, should be expressed by units of energy, mass, or quantity of electricity per "piece", i.e., per particle. Although the dimension "particle" is usually not introduced into the designations of the relevant units, it is present in latent form in the measurement of a number of quantities. Obviously, the first of such quantities is simply the total number of particles in a certain volume or mass.

Concentration. By relating the number of particles to a unit of volume, we obtain a quantity called concentration. Its dimension is

$$[n] = L^{-3} \tag{4.103}$$

and its units are m^{-3} and cm^{-3}. Introducing the dimension "particle" mentioned above, we can write

$$1 \text{ particle/m}^3 = 10^{-6} \text{ particle/cm}^3$$

In chemistry concentration is measured not by the number of particles, but by the number of kilomoles or moles per unit of volume. If the concentration in kilomoles per cubic metre or in moles per cubic centimetre is known, the concentration of particles can be determined if we multiply

the former concentrations by Avogadro's number, expressing in the first instance the number of particles per kilomole, and in the second the number of particles per mole. Most frequently concentration is measured in moles per litre. A concentration of 1 mole/l is called the *normal concentration.*

Diffusion coefficient. When its density or concentration is not uniform, a liquid, gas or dissolved substance will diffuse in a direction opposite to the density gradient or concentration, the amount of substance Δm diffusing during the time Δt being determined by the formula

$$\Delta m = -D\frac{d\rho}{dl}A\Delta t \tag{4.104}$$

where $\frac{d\rho}{dl}$ = density gradient

A = surface through which diffusion is taking place

D = diffusion coefficient.

The equation can also be written as

$$\Delta N = -D\frac{dn}{dl}A\Delta t \tag{4.105}$$

where ΔN = number of molecules that have diffused

$\frac{dn}{dl}$ = gradient of concentration.

Both formulas are identical, since the first of them can be obtained from the second by multiplying both sides by the mass of a molecule. Each of these formulas can serve for determining the diffusion coefficient. It is measured by the mass or number of molecules that diffuse in a unit of time through a unit of surface area with a density or concentration gradient equal to unity. Either of the formulas (4.104) and (4.105) gives the same dimension of the diffusion coefficient

$$[D] = \frac{[m][l]}{[\rho][A][t]} = L^2T^{-1} \tag{4.106}$$

or

$$[D] = \frac{[N][l]}{[n][A][t]} = L^2T^{-1} \tag{4.107}$$

which, as indicated above, coincides with that of kinematic viscosity. In the SI and mk(force)s systems the unit of the diffusion coefficient is m^2/s, and in the cgs system cm^2/s.

Distribution functions. The statistical nature of molecular processes appears in that the quantities characterizing the behaviour of molecules and other atomic particles are not the same for all the particles in a given system, but have the most diverse values distributed according to a definite law. Shown in Fig. 13 as an example is the maxwell velocity distribution of molecules. The hatched area under the curve between the values of the molecule velocities v_1 and v_2 shows the number of molecules whose velocities are greater than v_1 and smaller than v_2. The number of molecules in the small interval between the velocities v and $v + dv$ is

$$dN = F(v)\,dv \qquad (4.108)$$

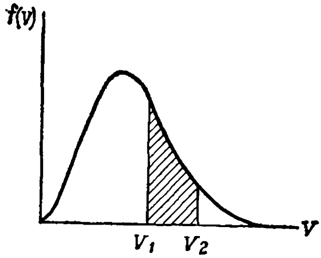

Fig. 13

where $F(v)$, being the ordinate of the curve, is called the function of the velocity distribution of the molecules. By definition,

$$F(v) = \frac{dN}{dv} \qquad (4.109)$$

The dimension of the distribution function by velocities is

$$[F(v)] = (\text{particle})\, L^{-1}T \qquad (4.110)$$

The functions of distribution by any other statistical characteristic—energy, length of free path, etc.,—can be determined in the same way as the function of distribution by velocities.

It can easily be seen that all these distribution functions have a dimension that is the ratio of the dimension "particle" to the dimension of the quantity, the distribution according to which is characterized by the given function.

The distribution function is often related to the total number of particles. A distribution function determined in this way is called "normalyzed per unit". Designating this function $f(v)$, we can write

$$f(v) = \frac{1}{N} F(v) \qquad (4.111)$$

where N is the total number of particles. Obviously

$$\int_0^\infty f(v)\,dv = 1 \tag{4.112}$$

It is easy to see that the dimension of the normalized distribution function is simply the reciprocal of the dimension of the quantity, the distribution according to which is determined by the given function.

CHAPTER FIVE

THERMAL UNITS

5.1. Temperature

The basic quantity in the science of heat is temperature. The concept of temperature is known to everybody from childhood. Moreover, it is "familiar" to every living creature, and even to every plant. Nevertheless or, perhaps, just for this reason it is very complicated to give a definition of temperature. In elementary textbooks temperature is sometimes defined as "the degree of heating of a body", sometimes as "the cause of feeling heat and cold". These definitions, while being illustrative to a certain extent, do not give a quantitative characteristic of temperature. Such a requirement can be met by strict definitions relating temperature to other thermodynamic functions. The latter, however, have another drawback—they are not so illustrative and require preliminary acquaintance with more complicated and abstract conceptions. For this reason, with a view to the objects of the present book, we shall proceed as follows: we shall assume that the reader is qualitatively acquainted with the concept of temperature and consider the problem of how to measure temperature. It does not have to be proved that everybody understands the terms "cold", "warm" and "hot", and also knows how to measure temperature with a conventional liquid thermometer.

It is easy to see, however, that with such measurements we cannot answer the question of how many times one temperature is greater or smaller than another one. According to the centigrade (one-hundred degree) scale used in everyday life we may have both positive and negative temperatures, so that the ratio between two temperatures may be either positive or negative, and may even be equal to infinity.

The "absolute temperature scale" (designated °K) introduced by W. T. Kelvin is quite widely known. In this scale all temperatures are positive, and the drawback mentioned above seems to disappear. The question nevertheless remains as to the extent to which the temperature measured according to the absolute scale is indeed "absolute", and what is the criterion that 600°K is twice as great as 300°K, or that the interval from 1 000°K to 1 500°K is five times greater than the interval from 400°K to 500°K. The matter is that although we do have the ability of feeling temperature (thermal feeling) and qualitatively comparring temperatures within a feasible range, we have at our disposal no methods for the direct measurement of temperature. To have an indirect method, we must relate temperature to other quantities whose measurement is accessible.

First of all we should turn to such properties of bodies surrounding us that according to our observations change with a change in temperature. It is natural here to make use of the expansion of bodies when heated. This gave birth to thermometers measuring the temperature according to the change in the volume of a liquid. Upon more thorough investigation it was found that an appreciable scatter of the results of measurements was hidden in this method, which can readily be illustrated. Assume that several thermometers filled with different liquids have been made. Let us mark the same "basic" or "fixed" points on them, for example, the melting points of two substances. Let us divide the scale between these points on all the thermometers into the same number of equal graduations. If all the thermometers are now placed in a medium having an intermediate temperature, the readings of different thermometers will be different. The thermometer that we decided to fill with water would behave especially queerly. At a temperature somewhat higher than the melting point of ice its column would be not higher, but lower than this point.

Thus, a different law of the change in the volume of different liquids with temperature (up to a change in the sign of the law) seems to deprive us of the possibility of finding a single-valued method of measuring temperature. Matters noticeably improved when Gay-Lussac discovered that all gases expand practically the same with a rise in temperature.

Mendeleev and Clapeyron succeeded in combining Gay-Lussac's law (an experimental one) with the experimental law of Boyle and Mariotte into a general law expressing the relationship between the volume of a gas and its pressure and temperature. Assuming that the volume of a gas at constant pressure or, more generally, the product of the volume of a given mass of gas and its pressure, is a linear characteristic of temperature, the combined law could be written as follows:

$$pV = C(1 + \alpha t) \tag{5.1}$$

where p = pressure of a gas
V = its volume
t = temperature read from any initial point
α = constant depending on the selection of the initial temperature point and the scale of measuring
C = factor depending on the mass of the given gas, the units of pressure and volume and the scale of measuring the temperature.

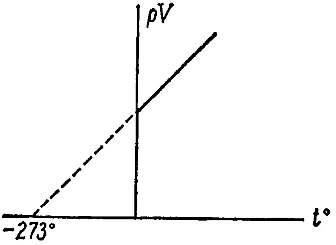

Fig. 14

Formula (5.1) can be depicted graphically by a straight line (Fig. 14) intersecting the axis of ordinates. It was found expedient to extrapolate the straight line to its intersection with the axis of abscissas and select the point of intersection as the beginning of the temperature scale. Thus the concept of "absolute temperature" was introduced. With respect to the scale for measuring this temperature, it could naturally be quite arbitrary. It was so selected as to divide the interval between the melting point of ice and the boiling point of water into 100 parts called degrees. With such a scale the point of intersection of the straight line with the axis of abscissas in Fig. 14 will be about 273 degrees to the left of the origin of coordinates. This point, as is known, was called "absolute zero". Formula (5.1) was transformed accor-

dingly, and it can now be written

$$pV = \frac{m}{M} RT \tag{5.2}$$

where T = absolute temperature
m = mass of a gas
M = mass of a kilomole or mole
R = so-called universal gas constant, whose numerical value depends on the selection of the units of the quantities in the formula.

In this form, together with Boyle-Mariotte's and Gay-Lussac's laws, equation (5.2) also includes Avogadro's law. This equation can in essence be interpreted as the definition of temperature as a quantity proportional to the product of the pressure and the volume of one mole of a gas.

Equation (5.2) makes it possible to measure the concentration of a gas by the so-called "reduced pressure". If the equation is rewritten as

$$\frac{m}{MV} = \frac{p}{RT} \tag{5.2a}$$

then the left-hand part will denote the number of moles or kilomoles per unit of volume, i.e., the molar concentration. The concentration will obviously be the same when at a temperature of $T_0 = 273°K$ the pressure of the gas will be

$$p_0 = \frac{p}{T} T_0 \tag{5.2b}$$

The pressure p_0 is called the reduced pressure, and it singularly determines the molar concentration and, consequently, the concentration of the molecules of a gas at a pressure p and temperature T. The corresponding relationship can easily be found if it is taken into consideration that 1 kilomole of a gas in standard conditions occupies a volume of 22.42 m^3. Thus, at a reduced pressure equal to one atmosphere, the molar concentration of a gas will be equal to 0.044616 kmole/m^3.

Knowing that one kilomole contains 6.023×10^{26} molecules, we find that such a concentration corresponds to 2.687×10^{25} molecules/m^3. The values of the molar concentration and the concentration of molecules at the reduced pressure expressed in various units are given in Table 18.

The development of the kinetic theory of ideal (perfect) gases made it possible to deduce equation (5.2) with a number of simplifying assumptions, including the one that the absolute temperature is proportional to the mean kinetic energy of translational motion of molecules. This relationship can be expressed by Boltzmann's formula

$$\bar{e} = \frac{\overline{mv^2}}{2} = KT \tag{5.3}$$

where K is a universal constant (not depending on the gas).

In the generally accepted form, equation (5.3) is written as

$$\bar{e} = \frac{\overline{mv^2}}{2} = \frac{3}{2} kT \tag{5.3a}$$

The constant k in this equation is called Boltzmann's constant.

Equation (5.3) makes it possible to give a definite physical meaning to temperature as a quantity proportional to the mean kinetic energy of the molecules. Such a definition of temperature, however, does not exhaust the possible relationships of temperature with other physical quantities. Let us consider some of them.

Suppose we have an enclosed envelope or shell isolated from the surrounding space and kept at a constant temperature, with an ideal vacuum inside. Notwithstanding the latter circumstance, it will not be absolutely "empty". The space within the shell will be filled with electromagnetic radiation whose radiant energy density e_r, according to the Stefan-Boltzmann law, is proportional to the fourth power of the absolute temperature of the shell

$$e_r = \sigma T^4 \tag{5.4}$$

where σ is a constant depending on the selection of the units.

The radiation inside the space is distributed by wavelength as shown in Fig. 15 for three different temperatures. As established by Wien, the wavelength of the maximum energy in this distribution is inversely proportional to the absolute temperature

$$\lambda_m = \frac{b}{T} \tag{5.5}$$

where b is a constant also depending on the selection of the units.

The two formulas (5.4) and (5.5) can be used for measuring and determining the temperature to the same extent as formulas (5.2) and (5.3). Such a determination of temperature from the formulas of radiation is even more general, since it can be used both for a space filled with a substance and for a vacuum. For this reason the widespread definition of temperature as a quantity proportional to the mean kinetic energy of translational motion of molecules should be considered as a particular one, namely, as the definition of the temperature of a body consisting of molecules, atoms and electrons. Quantum mechanics, however, limits even this definition, making it unsuitable for low temperatures. At the same time formula (5.4) is true for any conditions.

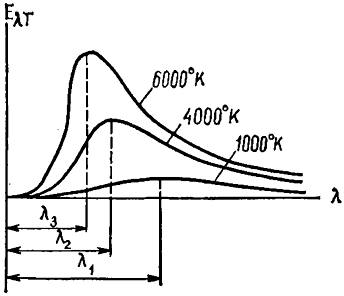

Fig. 15

According to the second law of thermodynamics, no heat machine, even the most ideal one, working without friction and heat losses to the surrounding medium, can have an efficiency (the efficiency η is the ratio of the useful work to the entire energy received by a system) equal to unity, since part of the heat must pass without fail from the heat source (heater) to the heat sink (cooler). If the temperature of the source is T_1, and that of the sink T_2, then the maximum efficiency of a machine (which, of course, cannot be achieved practically) is equal to

$$\eta = \frac{T_1 - T_2}{T_1} \tag{5.6}$$

If we define absolute zero as the temperature which a sink should have* for the ideal efficiency to equal unity,

* It should be noted that absolute zero cannot be reached in principle, but can be approached very closely.

we can use formula (5.6), only theoretically, of course, for constructing a temperature scale.

It is proved in thermodynamics that all the formulas given above determine the same temperature, which for this reason has been called the *thermodynamic temperature*. Any of the coefficients R, k, σ and b in formulas (5.2), (5.3a), (5.4) and (5.5) could, if desired, be assumed equal to unity, which would give different dimensions of temperature, viz., L^2MT^{-2}, $L^{-1/4}M^{1/4}T^{-1/2}$, and L^{-1}. Moreover, it would even be possible to change the definition of temperature itself, making it proportional not to the mean kinetic energy of translational motion of the molecules, but to the density of the radiation energy in an enclosed shell. Correspondingly all the formulas including temperature would change. For example, the product of the volume and the pressure of a gas, and the mean kinetic energy of the translational motion of molecules would be proportional to the fourth root of the temperature defined in this way. Naturally, such a step would lead to very appreciable inconveniences. It would also be inconvenient to replace temperature with a quantity proportional to it, for example, the product pV for one kilomole or mole of ideal gas, or the kinetic energy of one molecule, etc.

The exceedingly important place occupied by temperature in modern physics, since in a macroscopic system (i.e., in a system containing a large number of molecules and other particles) it determines most of its properties and the phenomena taking place in it (density, electrical conductivity, rate of chemical reaction, phase transformations, etc.), makes it expedient to relate temperature to the group of quantities having their own dimensions of their units, and accordingly it is desirable to include the unit of temperature among the basic ones. The designation of the dimension of temperature is θ.

According to the International system of units, the absolute temperature is defined as the thermodynamic temperature, a degree of this temperature being so established that the triple point of water has a temperature of exactly 273.16°K. By triple point is meant such a point at which all three phases of water, namely, ice, liquid water and saturated vapour are in equilibrium. While equilibrium

between two phases (water-vapour, ice-water, ice-vapour) is possible at different temperatures, the equilibrium of all three phases is possible only at a quite definite temperature, known as the *triple point*. According to the conventional scale of temperatures, the triple point of water is quite accurately equal to +0.01°C, so that the zero point of the conventional one-hundred degree scale corresponding to the melting point of ice at a pressure of 1 atm is equal to 273.15°K.

5.2. Temperature Scales

The absolute thermodynamic temperature scale (Kelvin scale) is used in scientific research to establish the relationships between temperature and other physical quantities. In everyday life, in engineering and even in laboratory practice use is made of the so-called centigrade or Celsius scale, named after Anders Celsius (it should be noted that the scale actually proposed by Celsius was inverted with respect to the one that bears his name, i.e., it had zero for the boiling point of water and 100 for the melting point of ice). Temperature measured by means of the Celsius scale is designated °C. For temperature intervals measured in degrees Celsius or Kelvin, use is made of the symbol *deg*, which also enters the designations of combined names of derived units.

In some countries the Réaumer scale is still in use (°R). In this scale the interval between the melting point of ice and the boiling point of water at a pressure of 1 atm is divided into 80 parts. In the Fahrenheit scale (°F) used in Great Britain and the USA, a temperature of 32°F is assigned to the melting point of ice, and of 212°F to the boiling point of water, so that this temperature interval is divided into 180 parts*.

We can now easily establish the relationship between the different temperature scales. Indeed, if we designate

* A scale in which the magnitude of a degree is the same as in the Fahrenheit scale, but where the temperature is counted from absolute zero, is called the Rankine scale. In this scale a temperature of 459.67° corresponds to 0°F, 491.67° to the freezing point of water and 671.67° to the boiling point of water.

the temperature interval between the melting point of ice and the boiling point of water by θ, then we obtain for one degree Kelvin or absolute (°K), Celsius (°C), Réaumur (°R) and Fahrenheit (°F) the following values:

$$°K = °C = \frac{\theta}{100}; \quad °R = \frac{\theta}{80}; \quad \text{and } °F = \frac{\theta}{180} \tag{5.7}$$

and consequently, any other interval Δt will be expressed by the values

$$\left.\begin{aligned} \Delta t°K &= \Delta t°C = \frac{\Delta t}{\theta}\,100 \\ \Delta t°R &= \frac{\Delta t}{\theta}\,80 \\ \Delta t°F &= \frac{\Delta t}{\theta}\,180 \end{aligned}\right\} \tag{5.8}$$

whence

$$\frac{\Delta t°K}{100} = \frac{\Delta t°C}{100} = \frac{\Delta t°R}{80} = \frac{\Delta t°F}{180} \tag{5.9}$$

or

$$\frac{\Delta t°K}{5} = \frac{\Delta t°C}{5} = \frac{\Delta t°R}{4} = \frac{\Delta t°F}{9} \tag{5.9a}$$

We stress the fact that the symbols $\Delta t°K$, $\Delta t°C$, $\Delta t°R$ and $\Delta t°F$ represent numbers measuring the same temperature interval in different degrees—temperature interval units. These numbers can be represented as the difference between the extreme temperatures of a selected interval measured according to the respective scale, in other words

$$\Delta t°K = t°K - t_0°K; \qquad \Delta t°C = t°C - t_0°C;$$
$$\Delta t°R = t°R - t_0°R; \text{ and } \Delta t°F = t°F - t_0°F$$

Taking $t_0°C = 0$ and, consequently, $t_0°C = 273°K$ *; $t_0°R = 0°R$; $t_0°F = 32°F$, we get

$$\frac{(t-273)°K}{5} = \frac{t°C}{5} = \frac{t°R}{4} = \frac{(t-32)°F}{9} \tag{5.10}$$

The last expression makes it very simple to convert temperatures from one scale to another.

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* Here 0°C is taken approximately equal to 273°K.

5.3. Fixed Temperature Points

The thermodynamic temperature scale defines temperature as a measured physical quantity and establishes its unit. The latter is taken as a basic one and is defined as follows: "the degree Kelvin is the unit of temperature according to the thermodynamic scale, in which the temperature of the triple point of water has the value of 273.16°K (precisely)". The word "precisely" denotes that this point is fixed as an unchangeable one.

In practice direct measurements in the thermodynamic scale are too complicated, and it is desirable to have the possibility of comparing various instruments serving to measure temperatures within comparatively narrow temperature intervals while retaining a sufficiently high accuracy. For this purpose use could be made of a gas thermometer, preferably a hydrogen or helium one, since these gases in comparison with others obey the laws of ideal gases to the highest degree. The use of a gas thermometer in practical conditions, however, is highly inconvenient, and for this reason several permanent fixed points were selected whose reproduction in laboratory conditions is not difficult. One of these points is given by the definition of the thermodynamic scale itself—the triple point of water, to which a constant temperature of 273.16°K is assigned. The remaining points have been established on the basis of measurements made as carefully as possible. All of them are temperatures of phase conversions at standard pressure (1 atm). These points are as follows:

Boiling point of oxygen	−182.97°C
Boiling point of water	100°C
Freezing point of zinc	419.505°C
Boiling point of sulphur	444.6°C
Freezing point of silver	960.8°C
Freezing point of gold	1063°C

5.4. Other Thermal Units

Quantity of heat. When speaking of the units of the quantity of heat, it should be noted first of all that the quantity of heat is in essence a measure of work, and not of energy, as is often thought. Indeed, if we subject a gas close to an

ideal one to isothermal expansion, then we shall have to impart to it a certain amount of heat, which will not be used to increase its "thermal" energy, but will be completely spent for performing external work. We have put the word "thermal" in quotation marks, since no specific thermal energy as a special form of energy actually exists.

Sometimes, in our opinion unsuccessfully, the term "thermal energy" is used with respect to the kinetic energy of the molecules of a substance. Heat received by a body, however, may be converted to a certain extent into internal energy of a system even at a constant temperature, if changes in the internal structure of the system occur, for instance, if a phase conversion takes place. The best example here is the melting of bodies that requires the addition of heat at a constant temperature. This heat is sometimes called "latent heat".

The first law of thermodynamics makes it possible to establish a measure of the quantity of heat according to the well known relationship

$$\Delta Q = \Delta U + \Delta W \tag{5.11}$$

where ΔQ = quantity of heat supplied to a system
ΔU = change in the internal energy of the system
ΔW = work done by the system to overcome external forces.

The quantity ΔU may include various kinds of energy, both an increase in the kinetic energy of various kinds of motion of the molecules (translational, rotational, oscillating) and a change in the energy of bond between separate molecules. It may also include the energy of dissociation, ionization, etc. Equation (5.11) shows that the quantity of heat can be measured in the same units as any kind of energy and any kind of work, including mechanical work. Hence, the dimension of the amount of heat

$$[Q] = L^2MT^{-2} \tag{5.12}$$

is the same as the dimension of work, and the units for measuring the quantity of heat should be the same as those for measuring work. Accordingly, in the SI system the quantity of heat is measured by the unit of work and energy—the joule.

Special units of the quantity of heat mentioned above (Sec. 4.4)—the *calorie* and *kilocalorie*—however, are also in great favour. These units were established in connection with calorimetric measurements of the quantity of heat with the use of a heat exchange process. Since the main substance used in comparing the quantity of heat was water, then the unit, the calorie, was correspondingly defined as the quantity of heat necessary for heating one gram of water by 1°C. The greater units established are the kilocalorie, equal to 1 000 calories, and the *therm*, equal to 10^6 calories. Accurate measurements, however, have shown that these quantities of heat are not constant and depend on the temperature interval used for heating. For this reason the mean calorie was introduced, which is defined as one-hundredth of the quantity of heat that must be imparted to one gram of water to heat it from its melting point to its boiling point. This calorie corresponds to heating water from 14.5 to 15.5°C.

When the equivalence of heat and work was established, special experiments were conducted to find the relationship between the units of the quantity of heat and work. These experiments determined the so-called "mechanical equivalent of heat"—a relationship according to which one kilocalorie is equal to 427 kgf-m.

With a view to the fact that a noticeable discrepancy exists between the values of a calorie or kilocalorie determined in different ways (calorimetrical, thermochemical), which led to the necessity of introducing corrections in accurate calculations, it was decided to do without determining the units of the quantity of heat by some kind of thermal measurements and establish a constant relationship between these units and the units of work, which was taken as follows:

$$1\ \mathrm{cal} = 4.1868\ \mathrm{J}$$

It is assumed that measurement of the quantity of heat by means of the calorie and its multiples and submultiples is to be retained as a temporary measure, and in the future it will be replaced with measurements using the unit of work of the SI system—the joule.

In conclusion it should be noted that in refrigeration engineering use is made of the concept "quantity of cold", which is in essence the quantity of heat that can be extracted by a refrigerating installation from the surrounding medium. The unit of the "quantity of cold" is the *frigorie*, numerically equal to one kilocalorie, but according to its definition having the reverse sign. It may be said that one frigorie is equal to minus one kilocalorie.

Temperature gradient. Similar to the pressure and velocity gradients considered previously, it is also possible to introduce the temperature gradient

$$\operatorname{grad} T = \frac{dT}{dl} \tag{5.13}$$

which with uniform distribution of the temperature can be written as

$$\frac{T_2 - T_1}{l_2 - l_1}$$

Its dimension is

$$[\operatorname{grad} T] = L^{-1}_{\text{мин}}\theta \tag{5.14}$$

and its units are deg/m and deg/cm if the temperature is measured according to the thermodynamic or one-hundred degree scale.

The *heat flow* (*heat flux*) is determined as the quantity of heat flowing in a unit of time in the direction of the temperature drop:

$$\Phi = \frac{dQ}{dt} \tag{5.15}$$

Its dimension

$$[\Phi] = \left[\frac{dQ}{dt}\right] = L^2MT^{-3} \tag{5.16}$$

coincides with that of power.

Depending on the unit used to measure the quantity of heat, the heat flow is measured in watts, kilowatts, megawatts, etc., or calories and kilocalories per second, minute or hour. The relationship between these units is given in Table 15.

The surface density of heat flow (*specific heat flow*) is the ratio of the heat flow to the cross-sectional area of the flow,

i.e., the flow per unit of cross-sectional area perpendicular to the direction of flow. According to the definition

$$q = \frac{d\Phi}{dA} \tag{5.17}$$

and its dimension is

$$[q] = MT^{-3} \tag{5.18}$$

Its units, accordingly, are equal to the units of heat flow related to one square metre or one square centimetre.

Entropy. Thermodynamics divides processes into reversible and irreversible ones. The former include isothermal and adiabatic changes in the state of an ideal gas. Ideal reversible processes, however, cannot be carried out in practice. All processes accompanied by friction, heat exchange, diffusion, etc. cannot be completely conducted in the reverse direction. Statistical physics connects this irreversible nature with a transition of the system from a less probable to a more probable distribution of the elements forming the system. We can consider as an example the process of mixing two gases that were initially separated in a certain vessel by a partition after the latter is removed. Another example is the levelling out of the temperatures of several bodies in contact that originally had different temperatures.

A quantitative measure has been established that allows us to judge of the degree of irreversibility of a process. This quantity is called *entropy* and designated S. If a system is transferred from a state that we shall designate by the subscript "1" to a state designated by the subscript "2", then according to the definition of entropy, its change in this process will be

$$\Delta S = \int_{1}^{2} \frac{dQ}{T} \tag{5.19}$$

It is proved in thermodynamics that

$$dS = \frac{dQ}{T} \tag{5.20}$$

is a total differential, so that the integral of dS around a closed contour is equal to zero. This means that entropy is a function of state. For a particular non-isolated system

the change in entropy may have any positive or negative value, and in particular be equal to zero. As follows from the second law of thermodynamics, however, in a closed system

$$\Delta S \geqslant 0 \tag{5.21}$$

The quantity ΔS characterizes the degree of irreversibility of the processes taking place in the given system.

Equation (5.19) determines the dimension of entropy

$$[S] = L^2MT^{-2}\theta^{-1} \tag{5.22}$$

and its units are J/deg, erg/deg, kgf.m/deg, cal/deg, etc.

5.5. Units of Thermal Properties of Substances

Heat capacity. Heat capacity is measured by the quantity of heat that must be imparted to a body to heat it by one degree. There are distinguished the *specific heat capacity*, or simply the *specific heat* (the quantity of heat required to heat one gram or kilogram), and the *molecular*, or *molar*, *heat* (the quantity of heat necessary for heating one mole or kilomole). The specific heat is found by the formula

$$c_{sp} = \frac{1}{m}\frac{dQ}{dT} \tag{5.23}$$

where m = mass of the body
Q = quantity of heat
T = temperature.

The dimension of specific heat is

$$[c_{sp}] = \frac{[Q]}{[m]\,[T]} = L^2T^{-2}\theta^{-1} \tag{5.24}$$

The relationship between the specific and molar heats is determined by the simple relationship

$$c_{mol} = c_{sp}M \tag{5.25}$$

Frequently use is made of the concept *volumetric specific heat*, which is the quantity of heat required for heating a unit of volume of a given substance by one degree. It is determined by the formula

$$c_{vol} = c_{sp}\rho \tag{5.26}$$

where ρ is the density of the substance.

The dimension of volumetric specific heat is

$$L^{-1}MT^{-2}\theta^{-1} \qquad (5.27)$$

The units of specific heat are J/(kg.deg), erg/(g.deg), cal/(g.deg), and kcal/(kg.deg).
Obviously 1 J/(kg.deg) = 10^4 erg/(g·deg); 1 cal/(g·deg) = = 1 kcal/(kg·deg) = 4.19 J/(kg·deg).

The units of molar heat are J/(kmole·deg), erg/(mole·deg), cal/(mol·deg) and kcal/(kmole·deg).

The relationship between these units is the same as between the corresponding units of specific heat.

The units of volumetric specific heat are J/(m^3·deg), erg/(cm^3·deg), cal/(cm^3·deg), kcal/(l·deg).

The relationship between them is 1 J/(m^3·deg) = = 10 erg/(cm^3·deg); 1 cal/(cm^3·deg) = 1 kcal/(l·deg) = = 4.19 J/(kg·deg).

Heat of transformation. When a substance passes over from one state of aggregation to another it is necessary to spend a certain quantity of heat at a constant temperature, called the heat of transformation (or transition). The latter, as heat capacity, can be related to a unit of mass, a mole or kilomole, or to a unit of volume. The corresponding dimensions differ from those of specific heat in the absence of the symbol of the dimension of temperature. In the same way the units of the heat of transformation differ from those of specific heat in the absence in their denominator of the unit of temperature interval—degree.

Heating value. Any fuel is characterized by its heating value, i.e., the quantity of heat that a certain amount of it can liberate upon combustion. The heating value, the same as the specific heat and the heat of transformation, can be related to a unit of mass, mole or kilomole, and to a unit of volume. The volumetric heating value is used exclusively for combustible gases, and is generally related to the volume of the gas taken in standard conditions ($t^\circ = 0^\circ$C and $p = 1$ atm). The units of heating value are the same as those of the heat of transformation.

Thermal conductivity. When there is a difference of temperatures in a medium, a heat flow will set in from the layer with the higher temperature to the one with a lower temperature. For a stationary univariate case this heat

flow can be expressed by the formula

$$\frac{dQ}{dt} = -\lambda \frac{dT}{dl} A \tag{5.28}$$

where $\frac{dQ}{dt}$ = heat flow

$\frac{dT}{dl}$ = temperature gradient

A = cross-sectional area of flow

λ = thermal conductivity of medium.

The unit of the thermal conductivity should be taken equal to the thermal conductivity of such a medium in which a heat flow equal to a unit of quantity of heat in a unit of time sets in through a unit of surface perpendicular to the direction of the flow at a temperature gradient equal to a unit of temperature over a unit of length. This definition and formula (5.28) give the dimension and units of the thermal conductivity:

$$[\lambda] = LMT^{-3}\theta^{-1} \tag{5.29}$$

The units of thermal conductivity are W/(m·deg), erg/(cm·s·deg), cal/(cm·s·deg), and kcal/(m·h·deg).

The relationships between them are

$$1\ \text{W/(m}\cdot\text{deg)} = 10^5\ \text{erg/(cm}\cdot\text{s}\cdot\text{deg)}$$

$$1\ \text{kcal/(m}\cdot\text{h}\cdot\text{deg)} = \frac{1}{360}\ \text{cal (cm}\cdot\text{s}\cdot\text{deg)}$$

Heat transfer coefficient. When there is a temperature difference ΔT at the boundary between two bodies, a heat flow sets in through this boundary that is determined by the formula

$$\frac{dQ}{dt} = \alpha \Delta T A \tag{5.30}$$

The coefficient α is called the heat transfer coefficient. It depends on the conditions at the boundary, in particular, at the boundary between a solid body and a fluid, and on the rate of flow of the latter. The heat transfer coefficient can be defined as the heat flow through a unit of boundary area at a temperature difference of one degree. Its dimension is

$$[\alpha] = MT^{-3}\theta^{-1} \tag{5.31}$$

The units of the heat transfer coefficient are W/(m$^2\cdot$deg), erg/(cm$^2\cdot$s$\cdot$deg), cal/(cm$^2\cdot$s$\cdot$deg) and kcal/(m$^2\cdot$h$\cdot$deg).
The relationships between them are:

$$1\ \text{W/(m}^2\cdot\text{deg)} = 10^3\ \text{erg/(cm}^2\cdot\text{s}\cdot\text{deg)}$$

$$1\ \text{kcal/(m}^2\cdot\text{h}\cdot\text{deg)} = \frac{1}{3.6}\times 10^4\ \text{erg/(cm}^2\cdot\text{s}\cdot\text{deg)}$$

Thermal diffusivity. Since the concept of thermal diffusivity is not always sufficiently well known, we shall deal with it in somewhat greater detail. Let us consider a homogeneous bar whose sides are perfectly thermally isolated, i.e., do not exchange heat with the surrounding medium. Let all the points of the bar originally have the same temperature T_0. If we now bring one of the ends of the bar into contact with a medium having the temperature T_1 (for purposes of determinacy let us assume that $T_1 > T_0$) then a heat flow will set in along the bar, and the temperature of all the points along it will begin to rise (Fig. 16). The curves t_0, t_1, . . ., t_∞ correspond to different consecutive moments of time.

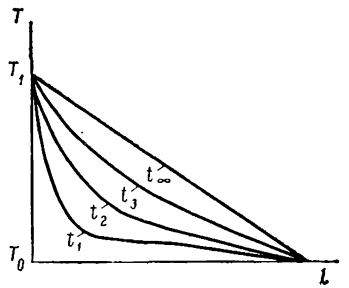

Fig. 16

Part of the heat passing through the bar will be spent for increasing the temperature of its different points, and a temperature gradient will appear along it. It is this process of the establishment of a temperature gradient that is called *thermal diffusion*. The process of thermal diffusion is obviously not stationary, since with a stationary heat flow through the bar the temperature gradient at all its points should be constant, not changing with time. The rate of change of the temperature at each point of the bar in the example described above (called a linear or univariate case) is determined by the equation

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial l^2} = a\frac{\partial\,\text{grad}\,T}{\partial l} \tag{5.32}$$

since

$$\frac{\partial^2 T}{\partial l^2} = \frac{\partial \operatorname{grad} T}{\partial l}$$

i.e., the derivative of the gradient along the axis is the change in the gradient per unit of length of the bar. The factor a is known as the thermal diffusivity and, as shown by theory, is related to the specific heat c_{sp}, the thermal coductivity λ and the density ρ as follows:

$$a = \frac{\lambda}{c_{sp}\rho} = \frac{\lambda}{c_{vol}} \tag{5.33}$$

Formula (5.32) defines the thermal diffusivity as the increase in temperature in a unit of time if the change in temperature gradient per unit of length is unity. A simpler definition is given by formula (5.33), according to which the thermal diffusivity is equal to the increase in temperature of a unit of volume of a given substance if it receives a quantity of heat numerically equal to its thermal conductivity.

The dimension of the thermal diffusivity

$$[a] = \frac{[\lambda]}{[c_{sp}]\,[\rho]} = \frac{LMT^{-3}\theta^{-1}}{L^2T^{-2}\theta^{-1} \cdot L^{-3}M} = L^2T^{-1} \tag{5.34}$$

coincides with that of the diffusion coefficient [see formula (4.106)]. This coincidence is not accidental. For a gas even the numerical values of the two quantities are quite close. This can be understood if we take into account that the kinetic theory of gases gives the following approximate relationship between the thermal diffusivity and coefficient of diffusion:

$$\lambda = D\rho c_v \tag{5.35}$$

where c_v is the specific heat of a gas determined at constant volume.

From formula (5.35) we get

$$D = \frac{\lambda}{\rho c_v}$$

i.e., in essence this is the same as formula (5.33). A stricter theory gives a factor of proportionality in formula (5.35) that differs somewhat from unity.

Temperature coefficients. Most of the physical properties of a substance depend on its temperature. If at a certain temperature T_0 the property we are interested in has the value A_0, then at a different temperature T this property will have the value A, which can be expressed in the form of the series

$$A = A_0 (1 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 + \ldots) \tag{5.36}$$

where $t = T - T_0$.

The coefficients α_1, α_2, α_3, etc. may have most diverse values, both positive and negative, and depend on the selection of the initial temperature T_0.

The absolute values of these coefficients often comply with the condition

$$1 > |\alpha_1| > |\alpha_2| > |\alpha_3| > \ldots$$

Frequently the coefficients α_2, α_3, etc. may be considered to be so small that

$$A = A_0 (1 + \alpha_1 t) \tag{5.37}$$

In particular, for the volume of a gas this gives Gay-Lussac's equation and, if $T_0 = 273°\text{K} = 0°\text{C}$, then, as is known, $\alpha_1 = 1/273$. Since the product $\alpha_1 t$ is an abstract quantity, then α_1 is measured in the units deg^{-1}.

The other coefficients, naturally, are measured in the units deg^{-2}, deg^{-3}, etc.

Coefficients of the van der Waals equation. The van der Waals equation of the state of a real gas has the form

$$\left(p + \frac{a}{V^2}\right)(V - b) = \frac{m}{M} RT \tag{5.38}$$

here p = pressure of the gas
V = volume occupied by the gas (volume of the vessel)
m = mass
T = absolute temperature
M = molecular weight
R = universal gas constant (see Sec. 5.1).

The quantities a and b, which are constant for a given mass of the given gas, have been introduced to take account of the forces of cohesion between the molecules and the

volume of the molecules themselves. The quantity

$$\frac{a}{V^2} = p_i \tag{5.39}$$

owing to the forces of molecular cohesion, has the dimension of pressure, and for this reason it is often (though unsuccessfully) called the internal pressure.

The units used to measure pressure and volume also determine the unit used to measure a. Since

$$V = \frac{m}{\rho}$$

and when $\rho = \text{const}$ $p_i = \text{const}$, then

$$a = V^2 p_i = \frac{p_i}{\rho^2}\ m^2 \tag{5.40}$$

i.e., a is proportional to the square of the mass.

If this constant for a kilomole or mole is designated a_0, then we can write

$$a = a_0\left(\frac{m}{M}\right)^2 \tag{5.41}$$

The constant b, proportional to the total volume of all the molecules, should be proportional to the mass of a gas, i.e.,

$$b = b_0\left(\frac{m}{M}\right) \tag{5.42}$$

where b_0 is the value of the constant for one kilomole or mole. The dimension of a from formula (5.40) is

$$[a] = L^5 M T^{-2} \tag{5.43}$$

The dimension of b is equal, naturally, to that of volume:

$$[b] = L^3 \tag{5.44}$$

The dimensions of a, a_0, b and b_0 determine their units. In practice a is frequently measured in atm/l^2 and b in litres.

CHAPTER SIX

ACOUSTIC UNITS

6.1. Objective Characteristics of Mechanical Wave Processes

In media having elasticity, mechanical deformations propagate with a velocity depending on the elastic properties and density of the medium. If the deformation is periodic, then waves propagate in the medium, their length being related to the frequency of oscillations ν and the velocity of propagation c by the equation

$$\lambda = \frac{c}{\nu} \tag{6.1}$$

As previously indicated, the frequency of oscillations is measured in hertz (Hz), and the wavelength in units of length—metres, centimetres, etc.

Oscillations whose frequency ranges from 16 Hz to 15-20 kHz are perceived by man's organs of hearing and are called *acoustic*, or *sound, oscillations*. Oscillations with lower frequencies are called *infrasonic*, and with higher ones—*ultrasonic*.

The characteristics of oscillations connected with the features of their psychophysiological perception are described in the next section. Here we shall deal with the quantities that have an objective nature and are determined by the corresponding general mechanical quantities that we already know. Although the SI system of units is recommended for use in all fields of science and engineering, in acoustics the cgs system is still in the greatest favour. The mk(force)s system is practically not used. Below are listed the most important quantities and their units in the SI and cgs systems.

Sound pressure. The appearance of sound oscillations in a gas or liquid is accompanied by oscillations of the

pressure of the medium. Thus the pressure at the given point at each given moment of time can be represented as the sum of the pressure in the non-excited medium, i.e., in the absence of oscillations, and a variable additional pressure called the *acoustic*, or *sound* pressure. During one period of oscillations the sound pressure changes its value and sign between the positive and negative amplitude values.

Sound pressure, as any other one, is measured in N/m^2 or dyn/cm^2. The latter was formerly called the bar in acoustics. But since the bar is now used to denote a pressure of 10^6 dyn/cm^2, the use of this name to denote a unit of 1 dyn/cm^2, has been discontinued, and the latter is now called a microbar (μb).

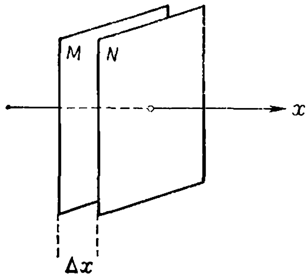

Fig. 17

Volumetric velocity. In a sound wave the particles of the medium oscillate with a velocity depending on the amplitude of oscillations, the frequency, and the phase. Assume we have a flat longitudinal wave (sound waves are longitudinal ones) propagating along the x-axis (Fig. 17). Let the particles of the medium in plane M have the velocity v at the given moment. Let us place plane N at a small distance Δx from plane M. During the time $\Delta t = \Delta x/v$ all the particles contained between M and N will pass through N. If we select an area A on plane N, then during the time Δt the volume $\Delta xA = v\Delta tA$ will pass through it, and during a unit of time the volume vA. This quantity is called the *volumetric velocity*. It is easily seen that its dimension and units are the same as those of the volumetric flow rate, i.e., m^3/s and cm^3/s.

Sound energy. Any volume of a medium in which waves are propagating has an energy consisting of the kinetic energy of the oscillating particles and the potential energy of elastic deformation.

Sound energy, as any other kind of energy, is measured in joules and ergs.

Density of sound energy. Sound energy related to a unit of volume of a medium is called the density of sound energy and is measured accordingly in J/m^3 and erg/cm^3.

Flow of sound energy. Waves spreading in a medium carry along with them a flow of energy. The energy conveyed in a unit of time through a given area perpendicular to the direction of propagation measures the magnitude of this flow. Obviously, the dimension and units of the flow of sound energy coincide with those of power, W and erg/s.

Sound intensity is the density of the flow of sound energy, i.e., the flow of energy related to a unit of surface perpendicular to the direction of the flow. The dimension of sound intensity is

$$[I] = MT^{-3} \tag{6.2}$$

The corresponding units are W/m^2 and $erg/(cm^2 \cdot s)$. The relationship between them is $1\ W/m^2 = 10^3\ erg/(cm^2 \cdot s)$.

Acoustic resistance. The amplitude of oscillations and, accordingly, the velocity of the oscillating points depend on the mechanical stress appearing in the medium, while for waves in a fluid on sound pressure. The instantaneous value of the velocity is determined by the relationship

$$v = \frac{p}{\rho c} \tag{6.3}$$

where p is the sound pressure and ρ the density of the medium.

If the left-hand and right-hand parts of equation (6.3) are multiplied by the area of the flow (for example, by the cross-sectional area of a pipe), then we can write

$$vA = \frac{p}{\rho c/A} \tag{6.4}$$

The quantity at the left is the volumetric velocity of oscillations. The ratio of the pressure to the volumetric velocity is called the acoustic resistance, since the appearance of formula (6.4) is the same as that of Ohm's law if the sound pressure is considered to be similar to the difference of potentials, and the volumetric velocity to the current intensity.

According to the definition of acoustic resistance, its dimension is

$$[R_a] = L^{-4}MT^{-1} \tag{6.5}$$

The units of acoustic resistance are N·s/m^5 and dyn·s/cm^5. The relationship between them is 1 N·s/m^5 = 10^{-5} dyn·s/cm^5

The name *acoustic ohm* is quite widely used for the unit dyn·s/cm^5, although it is not recommended by the relevant USSR State Standard.

In the general case the variable sound pressure and the variable volumetric velocity may not coincide in phase, and in such instances, similar to the impedance when dealing with alternating current, there is introduced the concept of complex acoustic resistance or acoustic impedance.

The acoustic resistance of a unit of surface area is called the *specific acoustic resistance* or the *acoustic resistivity* and is a characteristic of the given medium. It follows from formula (6.4) that the acoustic resistivity is equal to the product of the density of the medium and the velocity of propagation of oscillations

$$\eta = \rho c \tag{6.6}$$

The dimension of acoustic resistivity is

$$[\eta] = L^{-2}MT^{-1} \tag{6.7}$$

The resistivities of some media are given in Table 58.

Mechanical resistance. In addition to the acoustic resistance, it also becomes necessary in acoustics to deal with the so-called *mechanical resistance*, defined as the ratio between the periodic force and the velocity of oscillations. According to the definition

$$R_{mech} = \frac{pA}{v} \tag{6.8}$$

Its dimension is

$$[R_{mech}] = MT^{-1} \tag{6.9}$$

The units of mechanical resistance are N·s/m and dyn·s/cm. The latter unit is sometimes called the *mechanical ohm*.

Formula (6.9) determines the relationship between the units: 1 N·s/m = 10^3 dyn·s/cm.

Levels of sound intensity and sound pressure. For characterizing the quantities determining the perception of sound, of importance are not so much the absolute values of the sound intensity and sound pressure as their ratios to certain threshold values. For this reason there have been introduced the concepts of the relative levels of intensity and sound pressure. If the intensities of two sound waves are I_2 and I_1, then the logarithm of the ratio I_2/I_1 is called the difference between the levels of these intensities

$$L_1 = \log_{10} \frac{I_2}{I_1} \tag{6.10}$$

The unit of level difference is the *bel* (B), defined as the difference between the levels of two intensities whose ratio is equal to ten and, accordingly, the common logarithm of the ratio is unity. A tenth fraction of a bel, corresponding to the logarithm of a ratio equal to 0.1, is called the *decibel* (dB)*. With a level difference of 1 dB the ratio

$$\frac{I_2}{I_1} = 10^{0.1} = 1.259 \tag{6.11}$$

The difference in intensity levels measured in decibels is determined by the formula

$$L\,(\mathrm{dB}) = 10 \log_{10} \frac{I_2}{I_1} \tag{6.12}$$

In the same way as the difference in intensity levels, the difference in levels of sound energy flow (sound power) can be measured.

The following relationship exists between sound intensity and sound pressure:

$$I = \frac{p^2}{\rho c} = \frac{p^2}{\eta} \tag{6.13}$$

Consequently

$$\log_{10} \frac{I_2}{I_1} = 2 \log_{10} \frac{p_2}{p_1} \tag{6.14}$$

The method of measuring the difference in sound pressure levels is so established as to ensure that this difference will

* Other names have been suggested in recent years for the decibel, such as logit, decilit, decilog, decomlog and decilu (translator's note).

coincide with the difference in the intensity levels of the same oscillations. Accordingly, the difference in sound pressure levels measured in decibels can be found from the formula

$$L_p = 20 \log_{10} \frac{p_2}{p_1} \tag{6.15}$$

Together with measuring differences of levels in bels and decibels, they are also measured in *nepers* (Np). A difference in the levels of intensities of one neper corresponds to a ratio of the intensities equal to the base of natural logarithms. It follows from this definition that

$$1\ \mathrm{B} = 2.303\ \mathrm{Np} \tag{6.16}$$

The level of sound intensity and sound pressure is frequently related to a conditional threshold corresponding to a sound pressure of 2×10^{-5} N/m² or 2×10^{-4} dyn/cm²*.

6.2. Subjective Characteristics of Sound

The subjective perception of sound is characterized by a number of quantities that can be compared to a certain extent with some of the objective quantities considered above.

Pitch of sound. The main qualitative characteristic of a sound is determined by its frequency. We perceive different sounds as having equal intervals in pitch if the ratios of their frequencies are equal. Thus we can introduce the concept of *musical interval*, determined by the ratio of the frequencies of the sounds forming it. For example, an interval between sounds with frequencies of 200 and 500 kHz is equal to an interval between sounds with frequencies of 100 and 250 Hz.

A number of units constructed according to the logarithmic principle are used to measure musical intervals. The basic one is the *octave*, which is the interval between sounds with a frequency ratio of two. An octave is divided into 1 000 millioctaves or 1 200 *cents*. Another unit of musical interval is the *savart* (Sav) which is defined as an interval for which the common logarithm of the ratio of the frequencies of the sounds forming it is equal to 0.001. The magni-

* Logarithmic units are described in greater detail in Appendix 1.

tude of an interval measured in savarts is expressed by the formula

$$I_m = 1\,000 \log_{10} \frac{\nu_2}{\nu_1} \qquad (6.17)$$

The relationships between musical intervals and the ratios of the frequencies of sounds forming them are given in Table 22.

A series of tones with an interval of one octave between the first and last ones is called a *musical scale*. To get harmonic musical sounds, the separate intermediate steps of a scale—its tones—must have frequencies whose ratios form consecutive small integers. A scale whose tones satisfy this condition is called a *just*, or *natural*, scale.

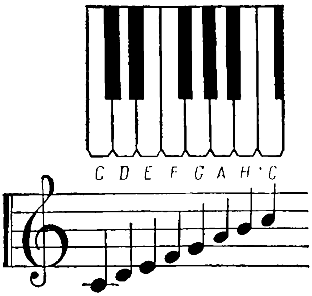

Fig. 18

For transition from one tonality or key to another, however, it is essential that it will be possible to form a new scale with the same ratios between the frequencies of the consecutive steps as in the basic scale, beginning from any tone. It is absolutely impossible to simultaneously meet both requirements in conventional musical instruments with the note system of recording music in general use. For this reason there was introduced a *tempered* scale in which an interval of one octave is divided into 12 semitones (halftones) with equal intervals between them. In accordance with the above, the interval between adjacent semitones is equal to 100 cents.

Table 23 gives the musical intervals forming the natural and tempered scales. Figure 18 shows part of a piano keyboard including one octave and gives the names of the respective keys and the notes corresponding to them. (In some countries the letter *B* is used instead of *H* to denote the seventh tone of the scale).

Timbre of sound. Different sounds even of the same pitch may differ from one another in their quality or timbre. The latter depends on the presence and relative intensity of additional oscillations, usually of higher frequencies

than the fundamental one determining the pitch of the sound. There are no quantitative parameters that could serve as a single-valued characteristic of timbre. In analysing musical tones, the relative intensity of the separate components is measured. In other words it can be said that timbre is determined by the kind of function of the distribution of the intensity of a sound by frequencies.

Loudness or volume of sound. Although the perception of a sound depends on its intensity, this relation, however,

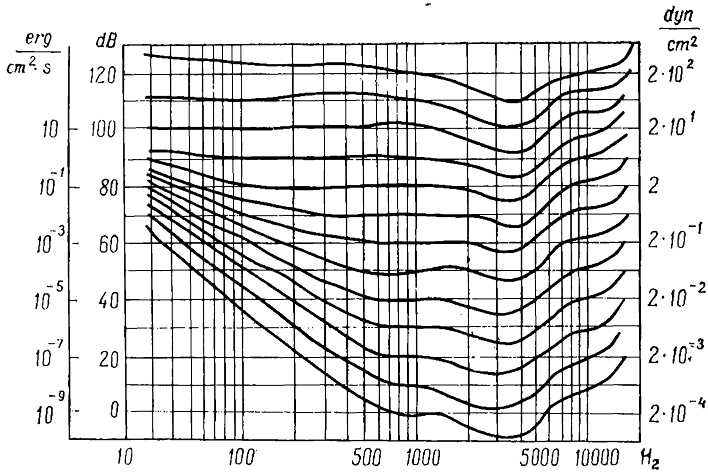

Fig. 19

is not a simple or single-valued one. First of all it should be noted that the sensitivity of the human ear to sounds of different frequencies is different. The lowest curve in Fig. 19 shows the so-called threshold of audibility—the minimum intensity of sounds of different frequency that the normal ear is capable of hearing. A logarithmic scale has been used in this figure along both axes—of abscissas and ordinates. The left-hand scale of ordinates indicates the intensities in erg/s·cm^2 and the levels of the intensities in decibels, the zero level being taken as the level of a sound of the minimum audible intensity at a frequency of

1 000 Hz. The right-hand scale of ordinates gives the corresponding sound pressures in dyn/cm^2.

The top curve corresponds to the appearance of mechanical perception that passes over into a feeling of pain. Upon an increase in the intensity of the sound of a given frequency, the feeling of loudness of the sound grows. The curves given in Fig. 19 are so constructed that the same loudness of sounds of different pitch heard corresponds to each curve. Thus, different levels of intensity correspond to sounds of equal loudness, but differing in frequency. The curves have been drawn in such a way that at a frequency of 1 000 Hz they are spaced 10 decibels apart. At other frequencies the difference in levels of adjacent curves is different.

Sounds are assumed to have equal intervals of loudness if the differences in the levels of sounds having the same loudness, but with a frequency of 1 000 Hz, are equal to 10 decibels. Since different intervals of the level of intensity correspond to equal intervals of the loudness level, a special unit, the *phon*, has been introduced for characterizing the level of loudness. The phon is defined as the difference in the loudness levels of two sounds of a given frequency, for which equally loud sounds with a frequency of 1 000 Hz differ in intensity by 10 decibels. By taking the level corresponding to the threshold of audibility as the zero one, we can directly measure the level of loudness of a sound in phons as the difference between the level of loudness of the given sound and the zero level.

All the units given above, constructed on a logarithmic basis, are, naturally, dimensionless ones.

6.3. Some Quantities Connected with the Acoustics of Buildings

When a sound wave impinges on a surface, part of the sound energy is reflected and part is absorbed. Correspondingly, the acoustic reflection and absorption factors are introduced. The *acoustic reflection factor* ρ is the ratio of the quantity of sound energy reflected from a large flat surface of uniform material to the quantity of energy incident on the surface during the same interval of time. The *acoustic absorption factor* α is equal to the difference between

unity and the acoustic reflection factor

$$\alpha = 1 - \rho \tag{6.18}$$

The *acoustic penetrance of a partition*, d, is the ratio of the quantity of energy passing through a partition to the quantity of energy incident on it during the same time.

All three quantities (ρ, α, and d) are dimensionless ones. The acoustic penetrance of a partition is determined by superposition of the processes of absorption in the substance which the partition is made of, multiple reflection from its front and rear surfaces, and partial passage through these surfaces. Account also has to be taken of the phenomenon of interference of waves that are superposed on one another in different phases. Pure absorption is observed if the thickness of the layer is so great that the intensity of the waves reflected from the rear wall can be neglected. If a plane wave is incident on the layer and its intensity after entering the layer is I_0, then at a certain distance x from the boundary of the layer the intensity will be

$$I = I_0 e^{-\delta x} \tag{6.19}$$

The factor δ is called the *linear absorption coefficient*. Its dimension is

$$[\delta] = L^{-1} \tag{6.20}$$

and its units are m^{-1} and cm^{-1}.

To characterize the absorption ability of individual bodies the concept of the *total absorbing power* of a body is introduced. It is determined by the product of the area of the body and its absorption factor. It is measured by the area of a perfect absorbing body having the same absorption as the given one. The unit of total absorbing power is the *square metre of open window*, since an opening in a wall practically reflects no sound.

Reverberation. When a sound is produced in a hall, the generated waves are repeatedly reflected from the walls, floor, ceiling and all the articles filling it. During each reflection part of the sound energy is absorbed, so that after the generation of oscillations is stopped the density of the sound energy at all the points gradually attenuates. If at the moment when the generation of sound is stopped the

density of the sound energy is u_0, then after the time t it becomes equal to

$$u = u_0 e^{-t/\tau} \tag{6.21}$$

The process of the production of a sound with its following decay is called *reverberation*. The characteristic time constant τ, as shown by W. Sabine, is equal to

$$\tau = \frac{4V}{c\Sigma\alpha A} \tag{6.22}$$

where V is the volume of the hall, and $\sum\alpha A$ the sum of the total absorbing powers of all the bodies in the hall, including the walls, floor, ceiling, furniture, people, etc.

The time τ is that during which the density of the sound energy decreases to $1/e$-th of the initial density. In practice a different quantity T is used, called the standard reverberation time, defined as the time during which the density of sound energy will decrease by 60 dB, i.e., 10^6 times. Since

$$10^{-6} = e^{-\frac{c\Sigma\alpha AT}{4V}}$$

we get

$$T = 2.3 \times 6\tau = 55.2 \frac{V}{c\Sigma\alpha A} \tag{6.23}$$

The reverberation time determines the acoustic properties of a room or hall. If this time is too short, the sounds will be muffled and "dull". If it is too long, the sounds will be superposed on one another and speech will become unintelligible. The optimal reverberation times depend on the designation of the premises and range from several tenths of a second to one or three seconds.

CHAPTER SEVEN

ELECTRICAL AND MAGNETIC UNITS

7.1. Introduction

The systems of electrical and magnetic units have gone through a complicated and, to a certain extent, contradictory period of formation, owing to the features of the development of our knowledge of electrical and magnetic phenomena. Up to the discovery by H. C. Oersted in 1820 of the magnetic action of an electric current, electrical and magnetic phenomena were studied independently, though by the same scientists (W. Gilbert, C. Coulomb, etc.). An appreciable part in the history of development of our knowledge of magnetic phenomena was played by the circumstance that man first became acquainted with them back in ancient times upon the discovery of the magnetic properties of iron.

When the time of quantitative investigation of electrical and magnetic phenomena arrived, then, owing to the external similarity between the interaction of permanent magnets and of electric charges, the same terminology was introduced for describing these interactions. This terminology has been retained up to the present time, although it does not correspond to our modern notions. Quite a few scientists, basing their investigations on the above-mentioned similarity, attempted to find the common nature of electrical and magnetic phenomena, but with no success. Although after the discovery made by Oersted and subsequent investigations it became clear that electrostatic and electromagnetic phenomena are quite different in nature, the description of these phenomena in courses on physics up to comparatively recent times was given in the following sequence: first electrostatics was studied, then magnetostatics, i.e., the science of interaction of permanent

magnets and their fields, next the laws of direct current, and only at the end of the course was the magnetic action of an electric current discussed. Magnetostatics also served as the basis for constructing the units of magnetic quantities, from which there were later formed the units of quantities characteristic of the magnetic action of electric current, electromagnetic induction, etc. Such a sequence of studying the material created difficulties for understanding the essence of phenomena and led to confusion in mastering the fundamentals of the subject.

At present most courses in physics use different methods of studying electromagnetism, in which a magnetic action of a current is taken as the basic magnetic phenomenon. This basis has also been used to introduce the unit of current intensity in the SI system—the ampere—which within the limits of this system is conditionally considered as a basic unit.

In this direction also, however, there is still no generally accepted method of setting out the course, and it will hardly be possible to indicate such a method. For this reason in the following section we shall consider different kinds of electrical and magnetic interactions and show how they can serve as the basis for constructing different systems of units.

7.2. Possible Ways of Constructing Systems of Electrical and Magnetic Units

Depending on the interactions and their form used to define the physical quantities serving to describe electrical and magnetic phenomena, a group of defining relationships is established by means of which the relevant derived units are introduced. In the mathematical expressions given below, that describe the quantitative aspect of the interactions, we shall, as previously, designate the factor of proportionality in the general form by the symbol C, regardless of its specific numerical value, and only when necessary will we supply it with a subscript.

Electrostatic interactions. Two electrically charged bodies are mutually attracted or repulsed with a force depending on the signs, magnitudes and distribution of the charges

on these bodies, their mutual arrangement, and the nature of the medium in which interaction occurs. In the general case, if charges of different signs are present on one or both bodies, a torque may appear in addition to the resultant force. The interaction will be the simplest if the bodies are small in comparison with the distance between them, and the charges may accordingly be considered as point ones. Here, assuming that the interaction takes place in a vacuum, the force of interaction can be written as the formula of Coulomb's law

$$f = C\frac{Q_1Q_2}{r^2} \tag{7.1}$$

Assuming in formula (7.1) that $f = 1$, $r = 1$, and $Q_1 = Q_2$ and, as usual, that $C = 1$ and has a zero dimension, we obtain the derived unit of charge (quantity of electricity), by means of which we can construct the units of all the quantities describing the properties of an electric field, conductors and dielectrics. The first of these quantities—the vector characteristic of an electric field—the *electric field intensity* E is measured by the force acting on a positive charge equal to unity, placed in the given field. By using this definition and writing it without the factor of proportionality in the form

$$E = \frac{f}{Q} \tag{7.2}$$

we can define the unit of electric field intensity as the intensity of such a field in which a unit of charge is acted upon by a force equal to unity. Further, by using the corresponding definitions, we can establish the units of other quantities such as potential, capacitance, and polarizability. The adopted unit of charge also makes it possible to establish the unit of current intensity by the formula

$$I = \frac{Q}{t} \tag{7.3}$$

according to which the intensity of an unchanging current is defined as the amount of electricity flowing through a cross section of a conductor in a unit of time. Strictly speaking, a factor of proportionality should also be used in formula (7.3), since an electric current is a new phenome-

non and its registration and measurement may not be connected with the measurement of an electric charge. Since in all systems, however, formula (7.3) is considered as a definition of current intensity, we have omitted this factor. It should be noted in passing that Abraham proposed a system of units in which the right-hand part of formula (7.3) contained a dimensional factor differing from unity.

By using Ohm's law, we can further determine the unit of resistance and thus construct an electrostatic system of units. By taking as the basic units the unit of length—centimetre, of mass—gram, and of time—second, we shall obtain a system that has been called the *electrostatic system* and is designated cgse (or esu for "electrostatic units"). Here the letter e is not a symbol of an additional basic unit, but only serves to show that the system is based on electrostatic interactions.

Interaction of permanent magnets. When investigating the interaction of magnets, Coulomb established that if long straight magnets are so arranged that the distance r between their poles is much smaller than their length, then the force of interaction between the poles is inversely proportional to r^2. Upon comparing this with the law of interaction of electric charges also discovered by him, he introduced the concept of *magnetic charge*, *quantity of magnetism* or *magnetic mass* m, which should be similar to an electric charge. By writing Coulomb's law for the interaction of magnets in a form similar to formula (7.1)

$$f = C\frac{m_1 m_2}{r^2} \qquad (7.4)$$

it was possible to establish a unit of magnetic mass similar to the way used to establish the electrostatic unit of the quantity of electricity. By analogy with the electric field intensity, there was introduced also the concept of the vector quantity of the magnetic field intensity H, determined by the force which a unit magnetic pole is subjected to in the given field, i.e., such a pole whose "magnetic mass" is equal to unity

$$H = \frac{f}{m} \qquad (7.5)$$

We indicated above that the similarity between electrostatic and magnetic interactions has a purely superficial nature that does not correspond to the essence of the phenomena. This, in particular, was revealed in the fact that, as we shall see below, the coincidence of the names "field intensity" for the vectors E and H does not conform with the part that these vectors play.

In the same way as the unit of electric field intensity was introduced in electrostatics, the unit of magnetic field intensity was introduced in magnetostatics as the intensity of such a field in which a pole whose magnetic mass is equal to unity is subjected to a force equal to unity.

Although the concept itself of magnetic mass was found to be completely fictitious, it was possible with its aid to establish the definitions of all the quantities describing a magnetic field and the magnetic properties of a substance, and construct a system of magnetic units on the basis of these definitions. This system, in which the basic units are also the centimetre, gram and second, has been called the *electromagnetic system* and is designated *cgsm* (or *emu* for "electromagnetic units"). Here the letter *m*, as the letter *e* in the designation of the cgse system, serves only to show that the system is based on magnetic interactions. The system has been given the name "electromagnetic" since it includes the units of quantities characteristic of the magnetic properties of a current, and the units of all the electrical quantities have also been constructed on its basis.

Electromagnetic interactions. Various experiments that are different in appearance, but have a common nature, can be conducted to show the magnetic field of an electric current. The first experiment of this kind was the deviation of a magnetic needle under the action of an electric current observed by Oersted. By determining experimentally what magnetic field of a permanent magnet causes the same deviation of the needle, Biot and Savart laid the foundation for establishing the law determining the magnetic field of a current. The difficulty involved in obtaining a general law on the basis of such experiments consisted in that whereas the interaction of electric charges could be studied on very small charged bodies that behaved as point ones, it was impossible in principle to create a "point current",

since any current must flow along a closed circuit. This difficulty was evaded by Laplace, who proposed a formula representing the magnetic field of any closed circuit as the geometrical sum of the fields created by separate elements into which the given circuit can be imaginarily divided. Laplace's formula (expressing the Biot, Savart and Laplace law) for an element of a current can be written as

$$dH = C\frac{I\,dl\sin\alpha}{r^2} \tag{7.6}$$

where dl = length of an element of the circuit
r = distance between this element and the point at which the field intensity is being determined
α = angle between r and dl (Fig. 20).

The direction of the vector dH, coinciding with the direction of the force acting on the north pole of a magnetic needle, is determined by one of the mnemonic rules, for instance, the right-hand screw rule.

To determine the magnetic field of a closed circuit having an arbitrary shape, formula (7.6) should be integrated around the entire circuit

$$H = C\oint\frac{I\,dl\sin\alpha}{r^2} \tag{7.6a}$$

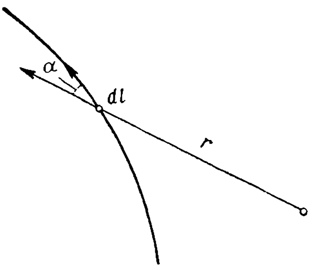

Fig. 20

Formula (7.6) gives several possibilities for selecting units. If we equate C to unity and measure current intensity in electrostatic units, we will have to introduce a new unit for magnetic field intensity and name it the electrostatic unit of magnetic field. All the remaining units characterizing a magnetic field will change accordingly. Another possibility is to measure magnetic field intensity using an electromagnetic unit, also with C equal to unity, and establish a new electromagnetic unit of current intensity.

Finally, we can retain both the electrostatic unit of current intensity and the electromagnetic unit of magnetic field intensity and determine experimentally or theoretically the numerical value (and, accordingly, the dimension)

of the factor C. This way will lead us to the construction of the so-called symmetrical or Gaussian system of units, which at present is in the greatest favour among physicists. It should be stressed here that all the systems listed above are cgs ones, since they are constructed on the same basic units—the centimetre, gram and second. In contrast to the cgse and cgsm system, the Gaussian system is designated cgs.

Attention must also be drawn to the following very important circumstance. If we accept the cgse system for both electrical and magnetic quantities, then a dimensional factor of proportionality will appear in the equations of magnetism. It will be found immediately if we consider Coulomb's magnetic law (7.4). Since the units of force and distance are established, while the unit of magnetic field intensity determined by formula (7.6) gives us, according to formula (7.5), a unit of magnetic mass different from that introduced above, then formula (7.4) can be retained only if the factor C differs from unity and, as can be easily seen, has a definite dimension. Upon establishing the electromagnetic unit for current intensity and, according to formula (7.3), for electric charge, we shall have to introduce into formula (7.1) a dimensional factor differing from unity. The numerical values and dimensions of the factors will be considered below.

It is quite obvious that the methods listed above do not exhaust all the possibilities of constructing systems of electrical and magnetic units, even if we remain within the limits of the same three basic units (cm, g, and s). Factors could be introduced into any of the formulas (7.2), (7.3) and (7.5). It is possible to construct such a system of units, for example in which, with a view to certain practical considerations, we shall establish a prototype of one of the electrical or magnetic units and use not three, but four basic units.

The electromagnetic phenomenon that is the opposite to the action of a current on a magnetic needle—the action of the field of a permanent magnet on an electric current—gives the same possibilities for constructing systems of electrical and magnetic units as the ones listed above. It is possible either to determine the electromagnetic unit

of current intensity if the unit of magnetic field intensity has been established, or determine the electrostatic unit of magnetic field intensity if the unit of current intensity has been established. In both instances we shall use Ampere's formula for the force acting on an element of current in a magnetic field:

$$df = CHI\,dl \sin (\widehat{H, dl}) \tag{7.7}$$

If we take, for instance, a straight conductor arranged in a magnetic field perpendicular to the direction of the lines of force of the field, and, if we assume that $C = 1$, then a unit of length of this conductor will be acted upon by a force equal to unity if: (a) the field intensity is equal to unity in the cgsm system and the current intensity is equal to an electromagnetic unit of current, or (b) the current intensity is equal to unity in the cgse system and the field intensity is equal to an electrostatic unit of intensity. If the current intensity is measured in cgse units, and the magnetic field intensity in cgsm units, then we shall have to introduce a dimensional coefficient C differing from unity.

A somewhat different way of establishing the units will appear if we abandon the use of the interaction of magnets or a magnet and a current and turn to the interaction of two currents. There are sufficient physical grounds to adopt this way as the basic one. The interaction of currents can be related with full right to the fundamental phenomena of nature, such as universal gravitation and the interaction of electric charges. At the same time the magnetic properties of iron and other ferromagnetic substances are inherent only in these substances and reflect the features of their structure. Ferromagnetism belongs to the most complicated phenomena and its explanation became possible only on the basis of consideration of the interaction of electrons from the viewpoint of quantum mechanics.

If we take the interaction of currents for constructing systems of electrical and magnetic units, we can write this interaction in different ways depending on the configuration and mutual arrangement of the currents. We can, for example, consider the interaction of very long straight conductors

of small plane circuits at a distance that is great in comparison with their linear dimensions, etc.

We shall use the expression for the mechanical moment acting on a small plane circuit in the field of any arbitrary circuit. This expression can be obtained from Ampere's formula if we consider the forces acting on separate elements of a closed circuit arbitrarily arranged in a homogeneous magnetic field. For a field created by an arbitrary circuit to be considered homogeneous, we select a small "trial" circuit, and for the moment acting on it we can write

$$M = C \oint \frac{I_1\, dl \sin \alpha}{r^2} I_2 A \cos \varphi \tag{7.8}$$

where I_2 = current of small circuit
A = its area
I_1 = current in arbitrary circuit that is a source of the field which is an external one with respect to the small circuit
φ = angle between direction of a normal to the area of the small circuit in the given position and in the position in which the moment is maximum.

Up to now we did not consider the question of the part played by the medium and included the parameter characterizing the properties of the medium in the factor C, or assumed that all the interactions occurred in a vacuum. Let us now write Coulomb's law and formula (7.8) in such a way that the properties of the medium in which interaction takes place will be shown in explicit form. To distinguish the factors of proportionality, we shall supply them with numerical subscripts

$$f = C_1 \frac{Q_1 Q_2}{\varepsilon_r r^2} \tag{7.9}$$

$$M = C_2 \mu_r \oint \frac{I_1\, dl \sin \alpha}{r^2} I_2 A \cos \varphi \tag{7.10}$$

The characteristics of the medium ε_r and μ_r are called, as is known, the relative permittivity (dielectric constant) and the relative magnetic permeability.

Since the concepts of the electric field intensity E, electric displacement (induction) D, magnetic induction B, and

magnetic field intensity H are introduced for describing electrical and magnetic phenomena, the following groups of equations can be substituted for equations (7.9) and (7.8).

Electrostatic interactions		Electromagnetic interactions	
$f = C_3 E Q_2$	(7.11)	$M = C_4 B I_2 A \cos\varphi$	(7.12)
$D = C_5 \frac{Q_1}{r^2}$	(7.13)	$H = C_6 \oint \frac{I_1\, dl \sin\alpha}{r^2}$	(7.14)
$E = C_7 \frac{D}{\varepsilon_r}$	(7.15)	$B = C_8 \mu_r H$	(7.16)

The sequence in which the above equations have been written is not accidental. Equations (7.11) and (7.12) relate to the mechanical action (force or moment) resisted by a charge or circuit in the given specific conditions with account taken of the influence of the medium. Equations (7.13) and (7.14) characterize the field of the charge Q_1 and current I_1 without account taken of the influence of the medium. Finally, the last two equations (7.15) and (7.16) relate the characteristics E and B of a field determining the mechanical action to the properties of the medium and through the quantities D and H to the charges and currents that are the sources of the field.

Thus, a certain analogy can be established between the following pairs of quantities:

E and B

D and H

ε_r and $1/\mu_r$

It shows that the characteristics of a magnetic field ha... been named unsuccessfully. The origin of these names is connected with the fact that they were introduced during the development of the science dealing with the properties of permanent magnets. Coulomb's magnetic law, with account taken of the influence of the medium, was written as

$$f = \frac{m_1 m_2}{\mu_r r^2} \tag{7.17}$$

The external contradiction between formulas (7.10) and (7.17) is explained by the circumstance that in magnetostatics it was assumed that the "magnetic masses" do not depend

on the properties of the medium. An analysis of this question has shown that the "magnetic masses" of poles so change with a change in the medium that if we denote the "magnetic mass" in a vacuum by m_0, then in a medium with a magnetic permeability of μ_r the "magnetic mass" will be m_0/μ_r, so that instead of formula (7.17) we can write

$$f = \mu_r \frac{\frac{m_{01}}{\mu_r} \frac{m_{02}}{\mu_r}}{r^2} \tag{7.18}$$

The group of equations (7.11) to (7.16) makes it possible to construct systems of units of electrical and magnetic quantities in a great variety of ways if we add equation (7.3) relating the current intensity to the charge. As has been noted above, we omit the factor of proportionality that could have been used in this equation, since in all systems it is taken equal to unity.

From among all the diverse possibilities of constructing systems that are given by the group of equations (7.11) to (7.16) and (7.3) we shall consider only those combinations of factors that are realizable in practice. It must first be noted that we cannot simultaneously manipulate with all the factors, since at least one of them is determined as the result of an experiment. T

The relative permittivity and relative magnetic permeability, according to their definitions, are so selected that in a vacuum $\varepsilon_r = \mu_r = 1$, and they are dimensionless quantities. In addition, in all systems $C_3 = 1$. As a result five factors remain, four of which we can deal with at our discretion. We could, if desired, arbitrarily establish the fifth factor too, but for this purpose, as will be seen below, the number of basic units must be reduced.

Let us first consider the alternatives of constructing systems with the basic units cm, g and s. In the cgs and cgse systems, $C_5 = C_6 = 1$. Here, as shown by experiment, the product of the factors

$$C_4 C_6 C_8 = C_2 = \frac{1}{c^2} \tag{7.19}$$

where c is the velocity of light in a vacuum. It can be seen that c has the dimension of velocity by comparing the dimensions of the quantities in the formulas given above.

In the cgse system the factors are selected as follows:

$$C_4 = C_6 = 1 \text{ and } C_8 = \frac{1}{c^2}$$

The factor C_8 in the cgse system is frequently designated μ_0. In the symmetrical cgs system

$$C_4 = C_6 = \frac{1}{c} \text{ and } C_8 = 1$$

and in the cgsm system

$$C_4 = C_6 = C_8 = 1$$

This selection of factors determines the unit of current intensity and the units of B and H. The unit of charge is also established correspondingly. By combining formulas (7.11), (7.13) and (7.15) and reverting to experiment, we find that

$$C_3 C_5 C_7 = C_1 = c^2 \tag{7.20}$$

As mentioned above, $C_3 = 1$. In addition, the factor C_5 is also taken equal to unity. Thus $C_7 = c^2$. This factor is generally designated $1/\varepsilon_0$.

All three systems, cgs, cgse and cgsm, can be combined into a single one if we assume that $c = 1$, for which purpose it is obviously necessary to reduce the number of basic units. Since the factor c is the velocity of light, this will be possible if we make one of the units, time or length, a derived one instead of a basic one. When discussing the question of the number of basic units in a system (Sec. 1.4) we indicated the arbitrary nature of this number and noted that beside the possibility of converting the unit of mass from a basic to a derived one (by simultaneously equating to unity the inertial and gravitational constants), a further reduction of the number of basic units is possible by equating to unity the velocity of light in a vacuum. Here we have seen directly that this is possible.

The electromagnetic theory of light developed by Maxwell made it possible to calculate the velocity of light on the basis of the general equations of the electromagnetic field (Maxwell's equations). Depending on the system used to write these equations (cgs, cgse or cgsm), the velocity of light

will correspondingly be equal to c, $1/\sqrt{\mu_0}$ or $1/\varepsilon_0$, which, of course, gives the same value. Should we construct a system using the coefficients ε_0 and μ_0, but in which neither of them is equal to unity in a vacuum (the SI system is such a system of units), then the velocity of light will be equal to $1/\sqrt{\mu_0\varepsilon_0}$.

Of all the three systems listed above, in the following we shall consider in detail only the cgs (symmetrical) system, turning to the cgse and cgsm systems when this will be essential in particular cases.

Here we shall only note that the units of electrical quantities (charge, field intensity, potential, current intensity, capacitance, resistance, etc.) of the cgs system coincide with the relevant units of the cgse system, while the units of magnetic quantities (induction, magnetic flux, inductance, etc.) coincide with those of the cgsm system.

Let us now turn to the construction of units of the electrical and magnetic quantities in the SI system. The main part in the creation of this system was played by the circumstance that widespread use was made in practical work in electrical and radio engineering and in physics of the so-called practical units such as the coulomb, ampere, volt and joule. When these units were established, however, they were not combined into a harmonious system that would allow their direct use for electrostatic and electromagnetic calculations. For this reason the problem appeared of introducing such factors of proportionality into the system that would make its use possible in all fields of the science of electricity and electromagnetism and, after combination with mechanical, thermal and other units, would create a system covering all the fields of physics and engineering. To make it possible to relate the practical units of electrical and magnetic quantities to the mechanical units having a ready unit of energy—the joule, and simultaneously comply with the requirement that the units of length and mass must be decimal system multiples or submultiples of the cgs units the following condition has to be observed:

$$1\ \mathrm{J} = 10^7\ \mathrm{erg} = 10^7\ \mathrm{g\cdot cm^2/s^2} = 10^a\ \mathrm{g}\ (10^b)^2\ \mathrm{cm^2/s^2}$$

Hence, it follows that

$$a + 2b = 7 \tag{7.21}$$

Systems have been proposed with various combinations of the exponents a and b, namely 10^7 g and 1 cm (Blondel's system), 10^{-11} g and 10^9 cm (Maxwell's system in which the coefficient μ_0 is equal to unity), etc. The greatest attention was attracted to the system proposed by Giorgi in which $a = 3$ and $b = 2$, i.e., 1 kg and 1 m. Both these units are convenient for practical work and are directly represented by international prototypes. Since the system was so formed that a new unit had to be introduced into it (any of the electrical or magnetic units, for example, the ampere, volt or ohm), two new factors would inevitably appear in the expressions for Coulomb's law and electromagnetic interaction instead of one that is present in each of the cgse, cgsm and cgs systems.

With respect to the dimensions of the relevant units, three possibilities existed here. One of the factors (in Coulomb's law or the law of interaction of currents) could be considered as a numerical factor deprived of a dimension and the system of dimensions constructed in the same way as in one of the two systems, cgse or cgsm, or one of the electrical or magnetic units could be considered as the basic one and the system of dimensions correspondingly constructed using not three, but four basic units*. It was the latter way that was adopted in constructing the system of dimensions of the SI system. One of its advantages is the simpler form acquired by the dimension formulas. As we have already learned, the fourth quantity, the dimension of whose unit is included among the basic ones, is the unit of current intensity, the ampere. In formula (7.25) used to determine the ampere, the constant μ_0 is considered to have a dimension, although its numerical value is fixed. If this quantity were considered as a dimensionless one (the dimensions of l and r cancel out), then the dimension of the unit of current intensity would be

$$[I] = L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1} \tag{7.22}$$

* Here, naturally, we do not take into consideration the dimensions of temperature and light intensity, that are not included in the dimensions of any of the electrical and magnetic units.

whence the dimension of charge would be

$$[Q]=L^{\frac{1}{2}}M^{\frac{1}{2}} \tag{7.23}$$

This dimension differs from that of charge in the cgs system [see formula (7.30)] by a factor whose dimension is the reciprocal of that of velocity. Obviously, the units of current intensity and quantity of electricity will have the same dimensions (7.22) and (7.23) in the cgsm system.

Since the so-called rationalized form of writing the equations of electromagnetism first proposed by O. Heaviside came into great favour in literature on electrical and radio engineering, this form was adopted in constructing the SI system. In the rationalized form the coefficient 4π is written in the denominators of the equations of the interaction of electric charges (Coulomb's law). As a result in a number of equations that are relatively more frequently encountered in practice, this coefficient disappears, and the equations acquire a more symmetrical form. This relates first of all to Maxwell's equations describing the electromagnetic field.

In accordance with the above, the following values of the factors of proportionality in equations (7.11) to (7.13) are used:

$$C_3=C_4=1 \text{ and } C_5=C_6=\frac{1}{4\pi}$$

The factor C_8 will be designated μ_0 in the following.

The product of the factors $C_4C_6\mu_0$ is so selected that when the current intensities I_1 and I_2 are measured in amperes, lengths in metres and areas in square metres, the moment will be measured in newton-metres (N-m). For convenience of calculation let us take instead of equation (7.10) the force of interaction of two parallel straight current conductors. This is also expedient because of the fact that the definition of the ampere adopted at the 9th Meeting of the International Weights and Measures Congress in 1948 is based on this interaction.

Let us determine the intensity of the magnetic field of an infinitely long straight current conductor on the basis

of the Biot, Savart and Laplace law (7.14):

$$H = \frac{1}{2\pi} \frac{I_1}{r} \tag{7.24}$$

Using this expression, we get from Ampere's formula, written with account of the influence of the medium, the force acting on a section of a conductor having a length l and a current I_2, parallel to a conductor with the current I_1:

$$f = \frac{\mu_0 \mu_r}{2\pi} \frac{I_1 I_2 l}{r} \tag{7.25}$$

If this force were written in the cgsm system (using the non-rationalized form of writing the equations) we would have the formula

$$f = \mu_r \frac{2 I_1 I_2 l}{r} \tag{7.25a}$$

When introducing the practical units, the ampere was defined as 0.1 of the cgsm unit of current intensity. Assuming that $l = r$ and $I_1 = I_2 = 1$ A $= 0.1$ cgsm, we get

$$f = 2 \times 10^{-2} \text{ dyn} = 2 \times 10^{-5} \text{ N}$$

The SI system takes this relationship as the definition of the ampere, without relating it to the cgsm unit. The exact definition of the ampere was given in Sec. 1.5.

Formula (7.25) now makes it possible to determine the value of the coefficient $\mu_0 = 4\pi \times 10^{-7}$. If, as is conditionally assumed in the SI system, the ampere is considered to be a basic unit, then the dimension of the coefficient μ_0 will obviously be

$$[\mu_0] = LMT^{-2}I^{-2} \tag{7.26}$$

where I is the symbol of the dimension of current intensity. The coefficient μ_0 is known as the *magnetic constant*.

Although the name of μ_0 — N/A²—follows from formula (7.25), the name H/m is generally used, where H is the symbol of the unit of inductance—the *henry*, which will be defined below.

It should be noted that in accordance with the definition, the number $4\pi \times 10^{-7}$ is taken as a precise one that must not be changed when more accurate measurements are made.

The constant factor C_7 in equation (7.14) can be determined in different ways. By substituting in the following the designation $1/\varepsilon_0$ for C_7, Coulomb's law can be written as

$$f = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{\varepsilon_r r^2} \tag{7.27}$$

As has been noted above, in the SI system the velocity of light in a vacuum is equal to

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{7.28}$$

This relationship is equivalent to the experimental determination of the factor C_2 given above [formula (7.19)]. Taking into account that $c \approx 3 \times 10^8$ m/s, we find that

$$\varepsilon_0 = \frac{1}{4\pi \times 10^{-7} \times 9 \times 10^{16}} = \frac{1}{4\pi \times 9 \times 10^9}$$

Its dimension is

$$[\varepsilon_0] = L^{-3} M^{-1} T^4 I^2 \tag{7.29}$$

Instead of the name of ε_0 following from formula (7.27), viz., $\frac{\text{A}^2\text{s}^2}{\text{m}^2\text{N}}$, the name F/m is used, where F is the symbol of the unit of capacitance, the *farad*.

Thus, using the approximate value, we can write

$$\varepsilon_0 \approx 8.85 \times 10^{-12} \text{ F/m}$$

This coefficient is called the electric constant. It should be noted that previously the coefficients ε_0 and μ_0 regardless of their numerical value (including μ_0 in the cgse system and ε_0 in the cgsm system) were correspondingly called the dielectric constant (permittivity) and magnetic permeability in a vacuum. These names are quite unsuccessful, and at present their use has been discontinued. They may sometimes be encountered in literature, however, especially in comparatively old books.

7.3. Units of the CGS System

We shall begin a detailed analysis of the units of electrical and magnetic quantities with the cgs (symmetrical or Gaussian) system. This sequence is justified, firstly, by historical considerations, since it was formed as a harmo-

nious system before the other ones, and secondly, by the fact that its construction is simpler and more consistent than that of the SI system, which will be considered in the following section, together with the relationships between the units of the two systems.

Electric charge (*quantity of electricity*). According to Coulomb's law, the unit of the quantity of electricity in the cgs system (we shall omit the words "cgs system" in the following definitions as self understood) is such a charge that interacts with an equal charge at a distance of 1 cm in a vacuum with a force of one dyne. From this definition the dimension of the unit is as follows:

$$[Q]=L^{3/2}M^{1/2}T^{-1} \tag{7.30}$$

Surface density of charge. The surface density of a charge is the quantity of electricity per unit of surface. According to the formula

$$\sigma=\frac{Q}{A} \tag{7.31}$$

we shall have a surface density of charge equal to unity with such a uniform distribution of the charge over the surface of a conductor when there will be a unit of charge per square centimetre.

The dimension of surface density is

$$[\sigma]=\frac{L^{3/2}M^{1/2}T^{-1}}{L^2}=L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1} \tag{7.31a}$$

Intensity of electric field. The corresponding unit follows from the definition of the field intensity

$$E=\frac{f}{Q} \tag{7.32}$$

where f is the force acting on the charge Q. The unit of the intensity of an electric field (*electric gradient**) is the intensity at such a point of the field in which a force equal to 1 dyne acts on a unit positive charge. The sign of the charge must be indicated because the field intensity is a vector, and its direction has to be stated in its definition. The formula

* The name "electric gradient" is based on the similarity between the field intensity and the potential (see below).

for the intensity also permits us to obtain its dimension, namely,

$$[E]=L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1} \tag{7.32a}$$

Electric displacement. If interaction occurs not in a vacuum, but in a certain medium, then the force of interaction will decrease ε_r times, where, as previously, ε_r is the relative permittivity of the medium. The product $\varepsilon_r E$ is called

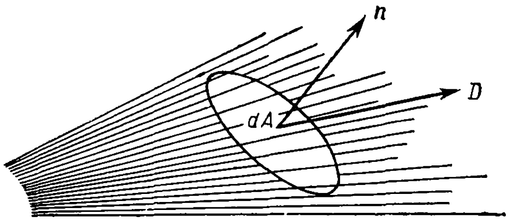

Fig. 21

the *electric displacement* or *electric induction* and is designated D. Since ε_r is dimensionless, the dimensions of D and E coincide.

Electric flux. The electric flux dN_d through an element of a surface dA is the product of the displacement, the area of the element, and the cosine of the angle between the direction of the displacement vector and a normal to the surface (Fig. 21):

$$dN_D = D\,dA\cos(\widehat{D,\, n}) \tag{7.33}$$

According to this definition, the unit of flux is the flux through 1 cm² of a surface perpendicular to the displacement vector at a displacement equal to unity.

Since according to the Gauss theorem the electric flux through any closed surface surrounding a charge Q is

$$N_D = 4\pi Q \tag{7.34}$$

it is obvious that the unit of electric flux is equal to the flux emitted from a charge equal to unity through a solid angle equal to one steradian. It follows both from the Gauss theorem and directly from the definition of the electric

flux that the dimension of the latter coincides with that of charge:

$$[N_D] = [Q] = L^{3/2}M^{1/2}T^{-1} \tag{7.35}$$

Potential. Potential is measured by the potential energy possessed by a unit of charge placed in the given point of a field. The unit of potential is the potential of such a point of an electric field in which a unit positive charge possesses potential energy equal to one erg

$$U = \frac{E_p}{Q} \tag{7.36}$$

The dimension formula of potential is

$$[U] = \frac{L^2MT^{-2}}{L^{3/2}M^{1/2}T^{-1}} = L^{1/2}M^{1/2}T^{-1} \tag{7.37}$$

The unit of potential can also be used, naturally, for measuring the difference of potentials, frequently called voltage.

In this instance the unit of voltage can be defined as the difference of potentials between two points, the transfer of a unit charge between which is accompanied by the performance of work equal to one erg.

The electromotive force (e.m.f.) of a current source is also measured in units of potential.

Dipole moment. A dipole is a system consisting of two egual charges opposite in sign, $+Q$ and $-Q$, placed a distance l apart. The *dipole moment* is the product of the magnitude of a charge and the distance between the charges. It is equal to unity if this product is also unity. From the formula for the dipole moment

$$\vec{p} = Ql \tag{7.38}$$

we get its dimension

$$[\vec{p}] = L^{5/2}M^{1/2}T^{-1} \tag{7.39}$$

The unit of the dipole moment can also be defined as the moment of such a dipole that is acted upon by a mechanical moment equal to unity in a homogeneous electric field with an intensity equal to unity, when it is arranged perpendicular to the field.

From the formula for the mechanical moment acting on a dipole

$$M = \vec{p}E \sin(\widehat{E, \vec{p}}) \tag{7.40}$$

we get the same dimension as above.

Capacitance. The ratio of the charge of a conductor to its potential determines the capacitance of the conductor

$$C = \frac{Q}{U} \tag{7.41}$$

The dimension of capacitance is

$$[C] = \frac{L^{3/2}M^{1/2}T^{-1}}{L^{1/2}M^{1/2}T^{-1}} = L \tag{7.42}$$

The unit of capacitance is the capacitance of a conductor whose potential increases by a unit of potential when a unit of charge is imparted to it.

Since the capacitance of a sphere in a vacuum is numerically equal to its radius, then the capacitance of a sphere with a radius of 1 cm can be taken as the unit of capacitance. For this reason the cgs unit of capacitance is often called "centimetre".

Dielectric polarization. A dielectric in an electric field is polarized, and each element of its volume is a dipole having a definite dipole moment. By dielectric polarization P is meant the dipole moment possessed by a unit of volume of a polarized dielectric. If the moment of the volume is $\vec{p}$, then

$$\vec{P} = \frac{\vec{p}}{V} \tag{7.43}$$

Correspondingly

$$[\vec{P}] = L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1} \tag{7.44}$$

Thus, the dimension of dielectric polarization coincides with that of field intensity and displacement.

The properties of a dielectric are characterized by two related dimensionless quantities—the *dielectric constant* (*relative permittivity*) ε_r and the *dielectric susceptibility* χ_e. The former was adequately discussed in the previous section.

The latter is determined as the ratio between the dielectric polarization and the field intensity

$$\chi_e = \frac{\vec{P}}{E} \tag{7.45}$$

The relationship between ε_r and χ_e is given by the formula

$$\varepsilon_r = 1 + 4\pi\chi_e \tag{7.46}$$

Current intensity. When charges move along a conductor, we have to do with current intensity, similar to the flow of a fluid or the heat flux and measured by the quantity of electricity flowing through a cross section of the conductor in a unit of time. In the general form the expression for current intensity is

$$I = \frac{dQ}{dt} \tag{7.47}$$

The unit of current intensity is that of a direct current at which a unit of charge flows through a cross section of a conductor in a second. According to this definition, the dimension of current intensity is

$$[I] = L^{3/2}M^{1/2}T^{-2} \tag{7.48}$$

Current density. The ratio of the current intensity to the cross-sectional area of a conductor is called the current density. The corresponding unit is a unit of current intensity per square centimetre. Its dimension is

$$[J] = \frac{[I]}{[A]} = L^{-\frac{1}{2}}M^{-\frac{1}{2}}T^{-2} \tag{7.49}$$

Resistance. According to Ohm's law, the current intensity is proportional to the difference of potentials across the ends of a conductor and inversely proportional to its resistance

$$I = \frac{U}{R} \tag{7.50}$$

The unit of resistance is the resistance of a conductor along which a current equal to a unit of current intensity flows with a difference of potentials across the ends of this conductor equal to a unit of potential. From the formula of

Ohm's law we get its dimension

$$[R] = L^{-1}T \tag{7.51}$$

Conductance. The reciprocal of resistance is called conductance:

$$G = \frac{1}{R} \tag{7.52}$$

This expression determines the unit of conductance and its dimension

$$[G] = LT^{-1} \tag{7.53}$$

Resistivity. The resistance of a homogeneous conductor with a constant cross section is expressed by the formula

$$R = \rho \frac{l}{A} \tag{7.54}$$

where l is the length and A the cross-sectional area of the conductor. The factor ρ characterizing the properties of the conductor is called resistivity. Its dimension is

$$[\rho] = T \tag{7.55}$$

The unit of resistivity is the resistivity of such a conducting material, each centimetre of whose length with a cross section of 1 cm^2 has a resistance equal to unity.

Conductivity. Similar to the definition of conductance, the conductivity σ is the reciprocal of resistivity. According to the definition,

$$[\sigma] = T^{-1} \tag{7.56}$$

Magnetic induction. The main characteristic of a magnetic field—the magnetic induction B—can be most clearly defined according to the mechanical action resisted by an electric current in a magnetic field. Let us use for this purpose formula (7.12) and assume in it that $\varphi = 0°$ and $A = 1$ cm^2. It should also be remembered that the factor of proportionality $C_2 = 1/c$. In these conditions the unit of magnetic induction can be defined as the induction of such a field in which the maximum moment acting on a circuit with an area of 1 cm^2 and through which there flows a current whose numerical value is equal to c (i.e., the velocity of light in a vacuum, measured in cm/s) is 1 dyn·cm. This unit of induction

is called the *gauss* (Gs). The gauss can also be defined as the induction of such a field in which each centimetre of a straight conductor arranged perpendicular to the field and along which a current of c units flows is acted upon by a force of one dyne. According to either of these definitions the dimension of induction is

$$[B] = L^{-\frac{1}{2}} M^{\frac{1}{2}} T^{-1} \tag{7.57}$$

Magnetic field intensity. The intensity of a magnetic field can formally be determined from formula (7.16) in which the factor C_8 is taken equal to unity, whence

$$H = \frac{B}{\mu_r} \tag{7.58}$$

With such a definition the unit of magnetic field intensity is the intensity of a field in a vacuum with an induction equal to one gauss. This unit is called an *oersted* (Oe). Since the relative permeability is a dimensionless quantity, the dimension of magnetic field intensity coincides with that of induction.

Magnetic flux. If a magnetic field is depicted by force lines whose density is proportional to the induction at the given point of the field, then the total number of lines of force penetrating the given surface can be characterized as the magnetic flux. The latter is determined by the product of the induction at the given point, an element of the area and the cosine of the angle between the direction of the vector of induction and a normal to the area

$$d\Phi = B\, dA \cos(\widehat{B, n}) \tag{7.59}$$

The unit of magnetic flux—the *maxwell* (Mx)—is the flux through an area of one square centimetre arranged perpendicular to a homogeneous magnetic field with an induction of 1 Gs. The dimension of magnetic flux is

$$[\Phi] = L^{3/2} M^{1/2} T^{-1} \tag{7.60}$$

If a magnetic flux penetrates a circuit containing a certain number of series-connected turns N, then sometimes use has to be made of the concept *flux linkage* Ψ, defined as the

product of the flux and the number of turns

$$\Psi = \Phi N \tag{7.61}$$

It is obvious that the dimension and unit of flux linkage are the same as those of magnetic flux.

Magnetic moment. Formula (7.12) expressing the moment acting on a circuit in a magnetic field includes the product of the current intensity and the area of the circuit. This product, that characterizes the given circuit and depends neither on the external magnetic field nor on the orientation of the circuit, together with a dimensional factor equal to the velocity of light determines the so-called magnetic moment of the circuit. According to this definition

$$\vec{p}_m = \frac{1}{c} IA \tag{7.62}$$

The magnetic moment is equal to the maximum mechanical moment acting on the given circuit when it is placed in a magnetic field with an induction of 1 Gs. The magnetic moment is a vector quantity. The direction of this vector is selected to coincide with a normal to the area of the circuit if, when looking along this normal, we see the current flowing clockwise through the circuit.

According to the definition of the magnetic moment, its unit is the magnetic moment of a circuit acted upon by a mechanical moment equal to 1 dyn·cm in a magnetic field with an induction of 1 Gs. Introducing the angle between the vector of induction and that of magnetic moment, we can rewrite formula (7.12) as follows:

$$M = \vec{B} p_m \sin (\widehat{B, \vec{p}_m}) \tag{7.63}$$

The dimension of magnetic moment is

$$[\vec{p}_m] = L^{5/2} M^{1/2} T^{-1} \tag{7.64}$$

The concept of magnetic moment is applicable not only to a circuit with a current, but also to a permanent magnet. In the chapter devoted to the units of atomic physics we shall also acquaint the reader with the magnetic moments of elementary particles.

Magnetomotive force (circulation of magnetic field intensity). According to the law of the total current, the integral

along a closed circuit of the scalar product $H\ dl$, where dl is an element of the circuit, is proportional to the algebraic sum of all the currents enclosed by the circuit:

$$\oint H\,dl\cos(\widehat{H,\,dl})=\frac{1}{c}4\pi\Sigma I \qquad (7.65)$$

The integral in the left-hand side is the circulation of the magnetic field intensity, which is generally called the *magnetomotive force* (*m.m.f.*) F. This name is connected with the previously mentioned erroneous analogy between the intensity of an electric field and that of a magnetic one. The circulation along a closed circuit of the electric field intensity caused by the action of external forces of non-electric origin is the electromotive force in the given circuit. It is equal to the work done to move a unit of charge along the circuit. The circulation of the intensity of a magnetic field is not connected with any motion or with any work, so that the name "magnetomotive force" is the same kind of anachronism as some other names that are still in use (horsepower, etc.).

Formula (7.65) is true both for homogeneous and heterogeneous media. With respect to the currents, such a direction is selected as the positive one that forms an angle less than 90° with the positive direction of a normal to the selected circuit.

The dimension of magnetomotive force follows from formula (7.65):

$$[F]=L^{1/2}M^{1/2}T^{-1} \qquad (7.66)$$

The unit of magnetomotive force—the *gilbert* (Gb)—is defined as the magnetomotive force when passing once around a conductor along which there flows a current of $c/4\pi$ units. The concept of magnetomotive force is used in calculating magnetic circuits. If we imagine a toroid (a short-circuited solenoid) with a cross-sectional area A containing N turns, then the magnetomotive force along the centre line of the toroid will be $\frac{1}{c}\ 4\pi\ IN$, where I is the current flowing through the turns of the toroid. At the same time the circulation of the magnetic field intensity is equal to Hl, where l is the length of the centre line. Hence the field

intensity is $H = \frac{1}{c} \frac{4\pi IN}{l}$. Going over from the field intensity to the induction, we can determine the flux penetrating the toroid

$$\Phi = \frac{1}{c} \frac{4\pi IN}{\frac{1}{\mu_r} \frac{l}{A}} = \frac{F}{R_m} \tag{7.67}$$

where μ_r is the relative permeability of the medium filling the toroid. The quantity in the denominator, namely,

$$R_m = \frac{1}{\mu_r} \frac{l}{A} \tag{7.68}$$

is called the *magnetic resistance* (or *reluctance*), since formula (7.67) has the same appearance as Ohm's law. The dimension of reluctance is

$$[R_m] = L^{-1} \tag{7.69}$$

The unit of reluctance is that of a circuit in which the magnetomotive force creates a flux of 1 maxwell. The reciprocal of reluctance is called *permeance*.

Inductance and mutual inductance. Upon a change in the magnetic flux linked with a given circuit, an e.m.f. appears in the latter that is determined by Faraday's law

$$\mathscr{E}_i = -\frac{1}{c} \frac{d\Psi}{dt} \tag{7.70}$$

If we have to do with a toroid or, what is the same, with a solenoid whose length is quite great in comparison with its diameter, then, using formula (7.67), we can write for the flux linkage

$$\Psi = \frac{1}{c} \frac{4\pi IN^2}{\frac{1}{\mu_r} \frac{l}{A}} \tag{7.71}$$

Assuming that the toroid is filled with a medium whose relative permeability does not depend on the field intensity, we can write the following instead of formula (7.70):

$$\mathscr{E}_i = -\frac{1}{c^2} \frac{4\pi N^2}{\frac{1}{\mu_r} \frac{l}{A}} \frac{dI}{dt} \tag{7.72}$$

Formulas (7.71) and (7.72) relate to a particular case when the flux whose change originates the e.m.f. of induction has been created by the current in a toroid or long solenoid. In the more general case of a circuit of any shape with any number of arbitrarily arranged turns, it is possible, on the basis of the law of Biot, Savart and Laplace, to express the flux linkage with this circuit as

$$\Psi = \frac{1}{c} LI \tag{7.73}$$

where the factor L depends on the configuration and dimensions of the conductors forming the circuit, and on the medium filling it. This factor is called the *inductance of the circuit* (its previous name is the coefficient of self-induction). Substitution for Ψ in formula (7.70) of its value from formula (7.73) gives

$$\mathscr{E}_i = -\frac{1}{c^2} L \frac{dI}{dt} \tag{7.74}$$

In a more general form, if the inductance does not remain constant, we should write

$$\mathscr{E}_i = -\frac{1}{c^2} \left(L \frac{dI}{dt} + I \frac{dL}{dt} \right) \tag{7.75}$$

It follows from formula (7.71) that the inductance of a toroid or a long solenoid is equal to

$$L = \mu_r \frac{4\pi N^2}{l} A \tag{7.76}$$

Any of the formulas containing the inductance can be used to determine its dimension and unit

$$[L] = L \tag{7.77}$$

The unit of inductance can be defined as the inductance of a circuit linked with a flux of 1 maxwell when a current equal to c units flows through it. According to another definition, the unit of inductance is the inductance of a circuit in which there appears an e.m.f. of induction equal to unity when the current in the circuit changes by c^2 units per second. In accordance with its dimension, this unit of inductance is sometimes called centimetre.

If we have two circuits that are rather close to each other, then with a current flowing through one of the circuits, part of or the entire flux will be linked with the second circuit. A change in the current in the first circuit will cause an e.m.f. of induction to appear in the second one. The formula for the flux linkage in one circuit depending on the current in the other will have a form similar to formula (7.73):

$$\Psi_2 = \frac{1}{c} MI_1 \tag{7.78}$$

where M, in contrast to L, is called the *mutual inductance*. When a current flows through the second circuit, the flux linkage in the first one will correspondingly be

$$\Psi_1 = \frac{1}{c} MI_2 \tag{7.78a}$$

the mutual inductance in both instances being the same. It is clear from the above that the physical meaning of inductance and mutual inductance is the same, and they accordingly have the same dimensions and units.

Intensity of magnetization (*magnetization*). If a body is placed in a magnetic field, each element of its volume acquires a magnetic moment. If the body has ferromagnetic properties, then the magnetization may remain after the external source of the magnetic field has been removed. The magnetic moment per unit of volume is called magnetization:

$$J = \frac{\vec{p}_m}{V} \tag{7.79}$$

Its dimension is

$$[J] = L^{-\frac{1}{2}} M^{\frac{1}{2}} T^{-1} \tag{7.80}$$

The magnetic properties of a substance are characterized by the *relative permeability*, which has been defined previously and according to its definition is a dimensionless quantity.

The hysteresis properties of ferromagnetic materials are described by the *residual magnetic induction* or *remanence* B_r and the *coercive force* H_c, whose meaning will be clear

from Fig. 22. They are measured, naturally, in gausses (B_r) and oersteds (H_c).

Another characteristic of the magnetic properties of a substance—the *magnetic susceptibility*—is related to permeability. It is determined as the ratio of the magnetization to the field intensity

$$\chi_m = \frac{J}{H} \tag{7.81}$$

It is easy to see that χ_m is a dimensionless quantity. The relative permeability and magnetic susceptibility are related by the expression

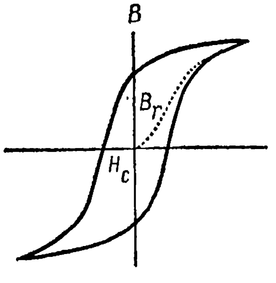

Fig. 22

$$\mu_r = 1 + 4\pi\chi_m \tag{7.82}$$

7.4. Units of the SI System

It was previously (Secs. 1.5 and 7.2) shown that the construction of a system of units which would include the practical units of current intensity, potential, charge, work, power, etc., can be carried out in different ways, by introducing an additional basic unit. When such a system, called the "absolute system of practical units", was developed, it was first intended to establish the unit of permeability as the fourth basic unit. Thus a fourth element would have to appear in the dimension formulas—the symbol of the independent dimension of the magnetic constant μ_0. In this system (mks μ_0), a development of the mks one, the dimension formulas of all the units correspondingly included the dimension symbols of length L, mass M, time T, and the constant μ_0. The inclusion of a fourth member in the dimension formulas was not novel, since previously within the limits of the cgs system (cgse and cgsm) a fourth member was also sometimes introduced—ε_0 in the cgse system and μ_0 in the cgsm one. It is quite obvious that in essence these systems, except for the appearance of the dimension formulas, do not differ in any way from the cgse and cgsm systems. It is clear that the dimen-

sion formulas in the mksμ_0 system completely coincide with those of the cgsm one.

Table 24, which gives the units of all the electrical and magnetic quantities in common usage in different systems, also gives the dimension formulas, the latter being presented in three forms: cgsε_0, cgs (Gaussian) and cgsμ_0. The dimensions in the cgse and cgsm systems can be obtained from the first and third ones if we omit the symbol ε_0 or μ_0 respectively.

As we already know, the unit of current intensity, the ampere, was selected as the fourth basic unit in establishing the SI system. Correspondingly the fourth element in the dimension formulas is the symbol of current intensity I. For this reason the dimension formulas in the SI system have a different appearance than those in the mks μ_0 one. This is the only difference between the two systems, since all the units in them are the same. With respect to the conversion of the dimension formulas from one system to the other, this can be done quite simply by substituting for the unit of the given system its expression in the other one in the corresponding dimension formulas. For purposes of illustration formula (7.83) gives the dimension of the unit of current intensity [which is a basic one (I) in the SI system] in the mks μ_0 system:

$$I = [I] = L^{1/2} M^{1/2} T^{-1} \mu_0^{-1/2} \tag{7.83}$$

If in all the dimension formulas of SI units given below the expression contained in formula (7.83) is substituted for I, then the dimension formulas in the mksμ_0 system will be obtained.

The relationship between the ampere and the unit of the cgs system can be established as follows. Two parallel currents, each with an intensity of 1 A, act on each other with a force of 2×10^{-7} N on a length equal to the distance between the conductors. When written in the cgs system this force of interaction will be

$$f = \frac{1}{c^2} \frac{2 I_1 I_2 l}{r} \tag{7.84}$$

Assuming that $I_1 = I_2 = I$ and $l = r$, and expressing the force of interaction in dynes (10^{-5} N) with $I = 1$ A, we get

$$2 \times 10^{-2}\ \text{dyn} = \frac{1}{c^2}\, 2I^2$$

Hence 1 A = $0.1c$ cgs units of current. Consequently we approximately have 1 A = 3×10^9 cgs units (designated cgs_I).

Electric charge (*quantity of electricity*). The unit of charge—the *coulomb* (C)—is defined, according to formula (7.2), as the quantity of electricity transported through the cross section of a conductor by a direct current with an intensity of one ampere. The relationship between the coulomb and the cgs unit of charge is obviously the same as between the relevant units of current intensity. The dimension of charge in the SI system is

$$[Q] = TI \tag{7.85}$$

It should be noted that for measuring the capacity of storage batteries the unit *ampere-hour*, equal to 3 600 °C, is used.

Potential (*difference of potentials, voltage, electromotive force*). For determining this unit let us use the formula for the power of a current

$$P = UI \tag{7.86}$$

According to this formula the unit of potential difference—the *volt* (V)—is defined as the difference of potentials across the ends of a conductor in which a power of 1 watt is liberated when a current of 1 ampere flows through it. Its dimension is

$$[U] = L^2MT^{-3}I^{-1} \tag{7.87}$$

From formula (7.86), assuming that 1 W = 10^7 erg/s and 1 A = $0.1c$ cgs_I, we get

$$1\ \text{V} = \frac{10^7}{0.1\,c} = \frac{1}{300}\ \text{cgs units } (\text{cgs}_U)$$

It should be noted in passing that in electrical engineering the name *volt-ampere* (VA) is frequently used instead of watt for measuring the "apparent power", i.e., the product $U_{ef}\, I_{ef}$ in the formula for the active power of an alternating current $P = U_{ef}\, I_{ef} \cos \varphi$.

Field intensity. The unit of field intensity can be determined either from formula (7.32), or from the expression for the field intensity of a point charge, or, finally, from the relationship between the field intensity and the potential

$$E = -\text{grad } U \tag{7.88}$$

Any of these definitions gives the following dimension for the unit of field intensity:

$$[E] = LMT^{-3}I^{-1} \tag{7.89}$$

The unit of field intensity does not have a special name. It can be called either *newton per coulomb* (N/C) or *volt per metre* (V/m), the latter being common usage. The non-system units *volt per centimetre* (V/cm), *kilovolt per centimetre* (kV/cm), etc. are in great favour. Obviously

$$1 \text{ V/m} = \frac{10^7}{10c} \cong \frac{1}{3 \times 10^4} \text{ cgs units } (\text{cgs}_E)$$

Displacement (*electric induction*). The electric field vector D is determined differently in the cgs and SI systems. It was shown above that in the cgs system the relationship between D and E is

$$E = \frac{D}{\varepsilon_r}$$

and, consequently, the dimensions of both vectors coincide. Matters are different in the SI system. Here E and D are related by the expression

$$E = \frac{D}{\varepsilon_0 \varepsilon_r} \tag{7.90}$$

and, accordingly,

$$[D] = [E][\varepsilon_0] = L^{-2}TI \tag{7.91}$$

Different dimensions of two quantities within the limits of a single system are an indication that these quantities have a different physical meaning. We would like to remind the reader that in general, whereas different quantities may sometimes have the same dimensions in one system or in different systems, quantities of the same physical nature can have different dimensions only in different

systems. For this reason the physical definitions of the vector D in the cgs and SI systems may differ, since the dimensions of E and D coincide in the cgs, and differ in the SI system.

Let us consider these definitions with the aid of the following example (Fig. 23). Suppose we have two identical plane capacitors connected in parallel that have been charged and disconnected from the source of voltage. The field

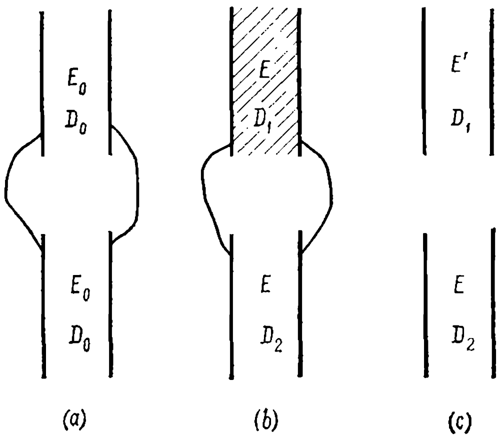

Fig. 23

intensities in both capacitors will naturally be the same, as will also be the displacements. Let the space between the plates of one of the capacitors be filled with a dielectric having a relative permittivity ε_r (Fig. 23*b*). It will be convenient to have a liquid dielectric for our further discussion. The difference of potentials between the plates of the capacitors will decrease, but remain the same for both capacitors since they are interconnected. For this reason the field intensity in both capacitors will also be the same. The charges on the capacitor plates, however, will now be different, and, accordingly, the values of the displacement vectors D will also be different. Assume that before connection of the capacitors the field intensities in them were E_0, and the displacement D_0. After introduc-

tion of the dielectric the field intensity will become E. The displacements in the capacitors will become $D_1 = \varepsilon_r E$ and $D_2 = E$ in the cgs system, and $D_1 = \varepsilon_0 \varepsilon_r E$ and $D_2 = \varepsilon_0 E$ in the SI system.

If we now disconnect the capacitors from each other and remove the dielectric from capacitor C_1, then the field intensity in it will grow ε_r times (Fig. 23*c*), while the displacement will not change. If the new intensity in this capacitor is E', then we can write:

in the cgs system

$$E' = D_1 \tag{7.92}$$

and in the SI system

$$E' = \varepsilon_0 D_1 \tag{7.92a}$$

Before removal of the dielectric, the field intensity E in capacitor C_1 may be considered to consist of the intensities of two fields—that of the charge on the plates (obviously equal to E') and that of the bound charges of the dielectric. After removal of the dielectric there remains only the field of the free charges on the capacitor plates.

Let us consider both capacitors before their disconnection as a single electrostatic system. We can now, within the limits of the cgs system, define the displacement vector as the field intensity of free charges (i.e., without taking into consideration the bound charges of the dielectric) with such an arrangement of these charges on conductors that is due to the presence of a dielectric. Indeed, according to expression (7.92), the displacement is the field of displaced charges whose redistribution between the capacitors was caused by the introduction of a dielectric into the capacitor.

To define the vector D in the SI system, let us introduce so-called "Mie plates" inside the dielectric, i.e., two small and very thin flat conductors first placed together. A charge will be induced on these plates whose density will depend on the value of D at the given point and on the orientation of the plates. The density of the charge will obviously be maximum when the plane of the Mie plates is perpendicular to the direction of the lines of force of vector D. In our example this direction is parallel to the capacitor plates,

and the density of the induced charge will be equal to that of the charge on the capacitor plates, since when the rationalized form of writing equations is used, the displacement in a plane capacitor is equal to the density of the charge on its plates

$$D = \sigma \tag{7.93}$$

The charge induced on the Mie plates can be measured if we first move them slightly apart and then remove them from the dielectric. In the general case of a non-uniform field the density of this charge, of course, will not equal the density of the charge on the conductors, but it will equal D, regardless of the distribution of the field. Thus, in the SI system, displacement can be defined as the maximum density of a charge induced on Mie plates in a given point of a field. The fact that the density of the induced charge depends on the orientation of the Mie plates (this is why the maximum density must be indicated) reflects the vector nature of displacement D.

The different nature of the physical definitions of the vector D leads to a number of inconveniences in setting out the course of physics and related subjects. Many scientists, including Academician M. A. Leontovich and professors I. G. Klyatskin and L. B. Slepyan objected to this division of concepts. Without going any deeper into their serious arguments, we shall only indicate that the homogeneous nature of the vectors E and D could be ensured in the SI system if we retained the relationship between E and D in the form $D = \varepsilon_r E$, and introduced the coefficient ε_0 in the expressions for calculating D according to a given distribution of charges. For example, for a point charge we should write $D = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ instead of $D = \frac{1}{4\pi}\frac{Q}{r^2}$.

The unit of displacement in the SI system and its relationship with the cgs unit can be obtained if we use any expression for D, for example, equation (7.93). According to the latter, the unit of displacement is the displacement in a plane capacitor with a charge density on its plates of 1 coulomb per square metre (C/m²). In the cgs system

$$D = 12\pi \times 10^5 \text{ cgs units } (\text{cgs}_D)$$

In conversion we have already used here the SI unit of surface charge density C/m^2, which is equal to 3×10^5 cgs units. The electric flux, defined as the product of the magnitude of the displacement vector, the area, and the cosine of the angle between the direction of the vector D and a normal to the surface (see Fig. 21), has a dimension coinciding with that of charge:

$$[N_D] = TI \tag{7.94}$$

The dielectric constant (*relative permittivity*) is defined in the same way as in the cgs system. Here, however, the following remark should be made. In literature on electrical engineering, besides the relative permittivity, use is made of the *absolute permittivity* ε, determined by the expression

$$\varepsilon = \varepsilon_0 \varepsilon_r \tag{7.95}$$

The dimension of absolute permittivity coincides with that of the electric constant ε_0 and its unit is also designated F/m

$$[\varepsilon_0] = L^{-3}M^{-1}T^4I^2 \tag{7.95a}$$

Dipole moment. The formula $\vec{p} = Ql$ determines its dimension:

$$[\vec{p}] = LTI \tag{7.96}$$

and its unit, coulomb-metre (C·m)

$$1 \text{ C}\cdot\text{m} = 3 \times 10^{11} \text{ cgs units } (\text{cgs}_{\vec{p}})$$

Dielectric polarization is the dipole moment of a unit of volume of a polarized dielectric

$$\vec{P} = \frac{\vec{p}}{V}$$

Its dimension is

$$[\vec{P}] = L^{-2}TI \tag{7.97}$$

and its unit is *coulomb per square metre* (C/m^2)

$$1 \text{ C/m}^2 = 3 \times 10^5 \text{ cgs units } (\text{cgs}_{\vec{P}})$$

Dielectric susceptibility is determined by the ratio $\chi_e = \frac{\vec{P}}{E}$. We shall obtain the relationship between χ_e and ε_r from the following expressions:

$$D = \varepsilon_0 E + \vec{P} = \varepsilon_0 E + \chi_e E = \varepsilon_0 \varepsilon_r E \tag{7.98}$$

and, consequently,

$$\varepsilon_0 (\varepsilon_r - 1) = \chi_e \tag{7.99}$$

or

$$\varepsilon_r = 1 + \frac{\chi_e}{\varepsilon_0} \tag{7.100}$$

Since the ratio χ_e/ε_0 has no dimension, then the dimension of χ_e coincides with that of the electric constant, and its unit is also designated F/m (farad per metre).

Upon comparing expression (7.100) with the similar expression in the cgs system

$$\varepsilon_r = 1 + 4\pi\chi_e$$

we shall find that the unit of dielectric susceptibility

$$1 \text{ F/m} = 9 \times 10^9 \text{ cgs units } (\text{cgs}_{\chi_e})$$

Capacitance. The unit of capacitance—the *farad* (F)—is the capacitance of a conductor whose potential increases by one volt when a charge of 1 coulomb is imparted to it. Since $C = \frac{Q}{U}$, its dimension is

$$[C] = L^{-2}M^{-1}T^4I^2 \tag{7.101}$$

The relationship between the units is

$$1 \text{ F} = \frac{0.1c^2}{10^8} = 9 \times 10^{11} \text{ cgs units } (\text{cgs}_C)$$

In practical work fractional units are generally employed, namely, the *microfarad* (μF) and the *picofarad* (pF).

Resistance. The unit of resistance—the *ohm* (Ω)—is the resistance of a conductor in which a current of one ampere flows with a difference of potentials of one volt across its ends. Ohm's law determines its dimension:

$$[R] = L^2MT^{-3}I^{-2} \tag{7.102}$$

It should be noted that the product RC has the dimension of time in both systems. In a circuit including a capacitor and a resistor, the product RC characterizes the time constant of charge attenuation. It is easy to see that

$$1\,\Omega = \frac{1}{9 \times 10^{11}} \text{ cgs units } (\text{cgs}_R)$$

Conductance. The unit of conductance is obviously the conductance of a conductor whose resistance is one ohm. This unit is called the *siemens* (S). In literature the name *mho* (reciprocal ohm) and the designation Ω^{-1} are sometimes encountered, although they are not recommended by the relevant standards. The dimension of conductance is the reciprocal of that of resistance

$$[G] = L^{-2}M^{-1}T^{3}I^{2} \tag{7.103}$$

Resistivity ρ is measured by the unit $\Omega \cdot \text{m}$

$$1\Omega \cdot \text{m} = \frac{1}{9 \times 10^{9}} \text{ cgs units } (\text{cgs}_\rho)$$

Its dimension is

$$[\rho] = L^{3}MT^{-3}I^{-2} \tag{7.104}$$

In practical work the resistivity is frequently measured in the units $\Omega \cdot \text{mm}^2/\text{m}$ and $\Omega \cdot \text{cm}$, which are obviously equal to 1×10^{-6} and 1×10^{-2} $\Omega \cdot \text{m}$, respectively.

The reciprocal of resistivity—*conductivity*—is measured by a unit that can be called *siemens per metre* (S/m).

Magnetic induction. The unit of magnetic induction—the *tesla* (T)—is the induction of such a field in which each metre of conductor with a current of one ampere and arranged perpendicular to the direction of the vector of induction is acted upon by a force of one newton. From this definition we get the dimension of induction

$$[B] = MT^{-2}I^{-1} \tag{7.105}$$

Substitution of the above units in the expression for induction connected with this definition, but written in the cgs system, gives

$$B = \frac{fc}{lI} = \frac{10^{5}\,\text{dyn} \times 3 \times 10^{10}\,\text{cm/s}}{100\,\text{cm} \times 3 \times 10^{9}\,\text{cgs}_I} = 10^{4}\,\text{Gs}$$

Thus

$$1\,\mathrm{T} = 10^4\,\mathrm{Gs}$$

Magnetic flux. The unit of magnetic flux—the *weber* (Wb)—is defined as the flux with an induction of 1 T through an area of 1 m^2 arranged perpendicular to the vector of induction. Hence its dimension is

$$[\Phi] = L^2MT^{-2}I^{-1} \tag{7.106}$$

and the relationship between the units is

$$1\,\mathrm{Wb} = 1\,\mathrm{T} \times 1\,\mathrm{m}^2 = 10^4\,\mathrm{Gs} \times 10^4\,\mathrm{cm}^2 = 10^8\,\mathrm{Mx}$$

Magnetic field intensity. To determine the unit of magnetic field intensity it will be convenient to make use of any of the corollaries of the Biot, Savart and Laplace law that give an expression for the intensity of the magnetic field of a current for specific circuits. Let us take for this purpose the formula for the intensity of the magnetic field at the centre of a circular current

$$H = \frac{I}{2R} \tag{7.107}$$

According to this formula the field intensity will be equal to unity if a current of 2 A flows around a ring with a radius of one metre, or, which is the same, a current of one ampere flows around a ring with a radius of 0.5 m. This unit has no special name. In accordance with its dimension in the SI system it is called *ampere per metre* (A/m). It was proposed to call this unit the *lenz* (in honour of E. Lenz).

The dimension of magnetic field intensity is

$$[H] = L^{-1}I \tag{7.108}$$

To establish the relationship between the units *ampere per metre* and *oersted* let us rewrite equation (7.107) in the cgs system:

$$H = \frac{1}{c}\,\frac{2\pi I}{R} \tag{7.109}$$

and insert the corresponding values, converting them to cgs units

$$1\,\mathrm{A/m} = \frac{2\pi \times 2 \times 3 \times 10^9}{3 \times 10^{10} \times 100} = 4\pi \times 10^{-3}\,\mathrm{Oe} \cong 1.26 \times 10^{-2}\,\mathrm{Oe}$$

In measuring the magnetic field of the earth, celestial bodies and outer space, a unit of magnetic field intensity called the *gamma* (γ) is used. It is equal to 10^{-5} Oe. Hence

$$1 \text{ A/m} = 1.26 \times 10^3 \ \gamma$$

It is worthwhile noting here that whereas in the cgs system the dimensions of the vectors B and H coincide, in the SI system they differ. A similar event occurred in electrostatics when we considered the vectors E and D. The objections that were raised against the lack of homogeneity of the vectors E and D observed in the SI system relate to an equal degree to the vectors B and H. This discrepancy could be eliminated quite easily if the magnetic constant μ_0 were introduced into the equation for the magnetic field intensity. If this were done the Biot, Savart and Laplace law could be written as

$$H = \frac{\mu_0}{4\pi} \oint \frac{I\, dl \sin \varphi}{r^2}$$

while the relationship between B and H would be the same as in the cgs system, i.e.,

$$B = \mu_r H$$

Magnetic moment. The unit of magnetic moment can be determined in two different ways, using either the expression for the mechanical moment acting on a circuit with current in a magnetic field, or the direct expression for the magnetic moment of a circuit. According to the first definition the unit of magnetic moment is the moment of a circuit that is subjected to a maximum torque of one newton-metre in a field with an induction of one tesla, and according to the second—the moment of a plane circuit with an area of one square metre through which a current of one ampere flows. Both definitions give the same dimension formula

$$[\vec{p}_m] = L^2 I \tag{7.110}$$

The unit of magnetic moment has no special name and is designated $\text{A}\cdot\text{m}^2$ (*ampere-square metre*). If we substitute for it an equivalent designation $\text{N}\cdot\text{m/T}$, it is easy to obtain

the relationship between the SI and cgs units

$$1\mathrm{A}\cdot\mathrm{m}^2 = 1\,\frac{\mathrm{N}\cdot\mathrm{m}}{\mathrm{T}} = \frac{10^5\,\mathrm{dyn}\times 10^2\,\mathrm{cm}}{10^4\,\mathrm{Gs}} = 10^3\,\frac{\mathrm{dyn}\cdot\mathrm{cm}}{\mathrm{Gs}}$$

Magnetomotive force. The circulation of the magnetic field intensity in the SI system is written as

$$F = \frac{1}{4\pi}\oint H\,dl\cos(\widehat{H,\,dl}) = \Sigma I \tag{7.111}$$

The unit of magnetomotive force is the circulation of the magnetic field intensity when a current of one ampere flows once around a circuit. The dimension of magnetomotive force coincides with that of current intensity and its unit is also called the ampere. Since when calculating magnetic circuits the total magnetomotive force is equal to the current intensity in each conductor multiplied by the number of turns, the magnetomotive force is frequently expressed in *ampere-turns* (At)

$$1\mathrm{A} = 1\mathrm{At} = \frac{4}{3\times 10^{10}}\times 3\times 10^9 = 1.26\,\mathrm{Gb}$$

Reluctance (*magnetic resistance*). The unit of reluctance is defined from the law of a magnetic circuit [formula (7.67)] as the reluctance of a magnetic circuit in which a magnetomotive force of 1 A creates a flux of 1 Wb. Formula (7.68) determines its dimension

$$[R_m] = L^{-2}M^{-1}T^2I^2 \tag{7.112}$$

The relationship between the units is

$$1\mathrm{A/Wb} = \frac{1.26\ \mathrm{Gb}}{10^8\ \mathrm{Mx}} = 1.26\times 10^{-8}\,\mathrm{Gb/Mx}$$

Inductance and mutual inductance. To determine the unit and dimension use can be made either of the expression for the relationship between the current in a circuit and the flux linking with it

$$\Psi = LI \tag{7.113}$$

or of the expression for the e.m.f. of inductance

$$\mathscr{E}_i = -L\,\frac{dI}{dt} \tag{7.114}$$

Equation (7.114) is written on the assumption that the inductance is constant. According to equation (7.113) the unit of inductance—the *henry* (H)—is defined as the inductance of a circuit that will be linked with a flux of 1 Wb when a current of 1 A flows through it. According to expression (7.114) the henry is the inductance of a circuit in which an inductance e.m.f. of one volt is induced when the current flowing through it uniformly changes by one ampere a second. Both definitions give the dimension

$$[L] = L^2 \mathrm{M} \mathrm{T}^{-2} I^{-2} \tag{7.115}$$

Comparison with formulas (7.73) and (7.74) written in the cgs system gives the relationship

$$1\ \mathrm{H} = 10^9 \text{ cgs units } (\mathrm{cgs}_L\text{—centimetres of inductance})$$

The same units are used to measure mutual inductance.

Intensity of magnetization (*magnetization*). According to formula (7.79), the unit will be such a magnetization when each cubic metre has a magnetic moment of one $\mathrm{A \cdot m^2}$. The name of this unit is accordingly *ampere per metre* (A/m) and coincides with that of the unit of field intensity. In the same way its dimension is

$$[J] = L^{-1} I \tag{7.116}$$

Upon rewriting formula (7.79) in the form

$$J = \frac{1}{c}\frac{IA}{V} \tag{7.117}$$

we can easily compare the ampere per metre with the cgs unit

$$1\ \mathrm{A/m} = \frac{3 \times 10^9 \times 10^4}{3 \times 10^{10} \times 10^6} = 10^{-3} \text{ cgs units } (\mathrm{cgs}_J)$$

It will be helpful to draw attention here to the following circumstance. Notwithstanding the fact that the units of magnetic field intensity and magnetization coincide in dimension and even in name, the relationship between these units and the corresponding cgs units is different. This is explained by the fact that in one case (field intensity) the equations are different in the rationalized and unrationalized forms, and in the other they are the same.

The given example once more illustrates the circumstance previously noted that the complex name of a derived unit can say nothing of its actual dimension if no indication is given of the specific defining relationship used to establish the given unit.

The magnetic properties of a substance—*relative permeability*, *remanence* (*residual magnetic induction*), and *coercive force*—require no special explanation. It should only be noted that in publications on electrical engineering, besides the relative permeability μ_r and the magnetic constant μ_0, there is also used their product

$$\mu_0\mu_r = \mu \tag{7.118}$$

called the *absolute permeability*.

Magnetic susceptibility. Equation (7.81) also defines magnetic susceptibility in the SI system. Since J and H have the same dimension, then χ_m, as in the cgs system, is a dimensionless quantity. The rationalized form of the equations, however, leads to the following relationship between μ_r and $\varkappa_m$

$$\mu_r = 1 + \chi_m \tag{7.119}$$

As a result the SI unit of magnetic susceptibility is $1/4\pi$ of the cgs unit.

7.5. On the So-called Wave Resistance of a Vacuum

In the previous section, using the example of the units of magnetic field intensity and of magnetization, whose dimensions and designations (A/m) coincide, an illustration was given of what was previously said about the absence of a single-valued relationship between the dimension formula of a unit and its concrete magnitude. This can be especially clearly illustrated by considerating the units and numerical values of a combined constant called the *wave* or *characteristic resistance of a vacuum*.

Upon the propagation of an electromagnetic wave in a medium with a relative permittivity and permeability of ε_r and μ_r, the amplitude and instantaneous values of the electric and magnetic field intensities obey the relationship

$$\sqrt{\varepsilon_0\varepsilon_r}\, E = \sqrt{\mu_0\mu_r}\, H \tag{7.120}$$

This expression can be written as

$$\frac{E}{H}=\sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} \tag{7.121}$$

The ratio E/H is generally called the *wave resistance* of a medium, since there is a formal analogy between equation (7.121) and Ohm's law. For a vacuum

$$R_x=\frac{E}{H}=\sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{7.122}$$

It is this quantity that is generally known as the wave or characteristic resistance of a vacuum. Let us consider the value of R_x in different systems of units. In the cgs system, where $\varepsilon_0 = \mu_0 = 1$ and have no dimensions, $R_x = 1$ and is also a dimensionless quantity. It should be remembered that in this system the dimension of resistance is $L^{-1}\,T$. In the cgse system $\varepsilon_0 = 1$ and $\mu_0 = 1/c^2$. In this system $R_x = 1/c$ and its dimension coincides with that of resistance. In the cgsm system $\mu_0 = 1$ and $\varepsilon_0 = 1/c^2$. Correspondingly $R_x = c$. The dimension of R_x coincides with that of resistance in this system too.

Let us consider, finally, the value of R_x in the SI system. Substitution of the values

$$\mu_0=4\pi\times10^{-7}\ \mathrm{H/m}$$

and

$$\varepsilon_0=\frac{1}{4\pi\times10^{-7}\times9\times10^{16}}\ \mathrm{F/m}$$

in equation (7.120) gives

$$R_x=120\pi\,(\mathrm{H/F})^{\frac{1}{2}} \tag{7.123}$$

Replacing H/m by V·s/A·m and F/m by A·s/V·m, we can write

$$R_x=120\pi\ \ \mathrm{V/A} \tag{7.123a}$$

Since the ratio volt/ampere defines the unit of resistance, the ohm, then it is assumed that the "wave resistance of a vacuum" is $120\pi = 377\ \Omega$.

If, however, we use the mks μ_0 system in its unrationalized form, in which the basic units are the same as in the SI system, and the unit of resistance, the ohm, is defined in the same way as the volt-ampere (since regardless of the form of writing the equations Ohm's law has the same appearance), then, taking into account that in this instance $\mu_0 = 10^{-7}$ H/m and $\varepsilon_0 = \frac{1}{9\times 10^9}$ F/m we find that

$$R_x = 30 \text{ V/A} \tag{7.123b}$$

i.e., 30 Ω.

The contradiction between the values of R_x determined in different ways is explained by the fact that the name of a complex unit is not at all a definition of this unit. In particular, in the example considered above the name V/A obtained as a result of the corresponding transformation of units or as the ratio of the units of electric and magnetic field intensities (V/m and A/m) cannot be interpreted as the unit of resistance. For this reason the concept "wave resistance of a vacuum" itself appears to be deprived of physical meaning, although it may sometimes be convenient to use one symbol for the expression $\sqrt{\mu_0/\varepsilon_0}$ in calculations. It should be noted here that if we relate the vectors E and B* instead of the vectors E and H, then instead of formula (7.120) we shall have

$$\sqrt{\varepsilon_0\varepsilon_r}\, E = \frac{B}{\sqrt{\mu_0\mu_r}} \tag{7.124}$$

or

$$E = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \frac{1}{\sqrt{\varepsilon_r\mu_r}} B \tag{7.125}$$

This expression, as can be easily seen, does not change when the units are changed.

7.6. International Units

As we have previously mentioned, the practical units that served as the basis of the SI system did not first form a single system, but made up an isolated group of units connected

* It will be useful to note that the relationship between the vectors E and B or, correspondingly, D and H is more logical from the viewpoint of Maxwell's equations.

to one another by several relationships. The introduction of these units played an important part in the development of the techniques of electrical and magnetic measurements, as a result of which soon after its appearance the practical system acquired international recognition. Much work was done to establish standards of the practical units of resistance, current intensity and potential difference; originally these standards or prototypes were intended to serve for reproduction of the ohm, ampere and volt, defined as 10^9, 0.1 and 10^8 of the respective units of the cgsm system. It was later found, as could have been foreseen, by the way, that there are insignificant, but nevertheless noticeable discrepancies between the established standards and their prototypes based on the absolute system. It was then decided, as was done previously with the metre and kilogram, to adopt the prototypes as legal international units of electrical quantities. These international units were defined as follows:

the *international ohm*—the resistance of a column of mercury 106.300 cm long with a mass of 14.4521 g and with an identical cross section along its entire length, measured at the melting point of ice and with an unvarying current;

the *international ampere*—the intensity of an unvarying current that deposits 0.00111800 grams of silver by electrolysis from a silver nitrate solution in one second;

the *international volt*—the electrical potential or electromotive force that induces a current of one international ampere in a conductor with a resistance of one international ohm;

the *international watt*—the power of an unvarying current of one international ampere at a difference of potentials of one volt.

The remaining international units, the same as the international volt and watt, are determined from the corresponding basic international units.

The considerably improved accuracy of electrical and magnetic measurements made it possible to perform the reverse transition and, having established in a definite way the exact formulation of one of the units (as we already know, the ampere), construct the mksμ_0 system of units that formed the basis of the SI system now adopted in many

countries. To distinguish them from the international units given above, the units of the mksμ_0 system were called "absolute" to underline the fact that they were constructed according to the same principle as those of the cgs system. To allow the results of measurements made in international units, during the time these units were still in use, to be converted to mksμ_0 units, the following relationships were established between them:

1 mean international ampere = 0.99985 abs. ampere
1 mean international ohm = 1.00049 abs. ohm
1 mean international coulomb = 0.99985 abs. coulomb
1 mean international volt = 1.00034 abs. volt
1 mean international henry = 1.00049 abs. henry
1 mean international farad = 0.99951 abs. farad
1 mean international weber = 1.00034 abs. weber
1 mean international watt = 1.00019 abs. watt

The adjective "mean" is due to the fact that when establishing the relationship between international and absolute units, it was found that there is a slight discrepancy between the prototypes of the international units kept in different countries, so that the mean value of these prototypes has been taken for purposes of comparison. For example, the following relationship existed between the international units adopted in the USSR and the mean international units:

1 USSR int. ohm = 1.000010 mean int. ohm
1 USSR int. volt = 1.0000075 mean int. volt

At present the international units have been completely discarded and replaced by "absolute" units, i.e., units of the SI system.

The definition of a basic unit of this system—the ampere—through mechanical units with the establishment of an exact value of the coefficient μ_0 in the defining relationship has made it possible to include the practical electrical and magnetic units into the general system of units of physical quantities.

CHAPTER EIGHT

UNITS OF RADIATION

8.1. Scale of Electromagnetic Waves

The field of investigated electromagnetic waves extends almost without interruptions from waves with a length of thousands of kilometres radiated by low-frequency electrical machines to the short-wave γ-radiation of radioactive elements and cosmic rays. Different ranges of this spectrum have different properties, propagate differently and manifest themselves in different ways. The narrow band of wavelengths from 0.38 to 0.76 micron is perceptible by our eyes; within certain ranges radiation is capable of calling forth chemical reactions, the photoeffect, and ionization of gases. Radiation with the greatest wavelengths can be detected with the aid of electromagnetic oscillating circuits. For this reason together with general characteristics of radiation, first of all from the viewpoint of its energy, there are specific characteristics for separate ranges of the spectrum of electromagnetic waves.

Wavelengths and the frequencies corresponding to them are measured in the usual units of length and frequency. It is quite natural that in the range of long waves the units of length are the metre and centimetre, while light and shorter waves are measured in microns, angstroms and X-units. Frequencies are generally measured in herz, while kiloherz and megaherz are used for radio waves. In addition to wavelengths and frequencies, in spectrometry use is often made of the *wave number* σ, which shows the number of waves per unit of length. Obviously

$$\sigma = \frac{1}{\lambda} \tag{8.1}$$

where λ is the wavelength. The wave number is frequently determined as

$$\sigma = \frac{2\pi}{\lambda} \tag{8.2}$$

The units of wave number are the reciprocal metre, reciprocal centimetre, reciprocal micron, etc.

8.2. Characteristics of Radiant Energy

Quantities characterizing the energy aspect of the radiation of electromagnetic waves are measured by the general units of energy used to measure energy, volume density of energy, energy flux, etc. The names of some of these quantities reflect the fact that they are the result of extended usage of the concepts employed in illumination engineering, although they may relate to such ranges of the spectrum that are not perceived by our eye.

Radiant flux (radiant energy flux) Φ_r is the amount of radiant energy passing in a given direction in a unit of time. Both with respect to its physical meaning and to its units and dimension, radiant flux is absolutely identical to energy flux considered in the chapter on acoustic units. We shall remind the reader that the units and dimension of energy flux coincide with those of power. It should be noted, however, that together with the units W and erg/s, the heat units cal/s, kcal/s, etc. are also used to measure radiant flux.

Radiant flux density (irradiance) $d\Phi/dA$ is the flux per unit of surface area. Here several concepts have to be distinguished, although their units and dimension coincide.

Energy flow S is the radiant flux in a given direction per unit of surface area perpendicular to the direction of radiation. The energy flow of electromagnetic waves is a vector (Poynting vector):

$$\mathbf{S} = (\mathbf{E} \times \mathbf{H}) \text{ (SI)} \tag{8.3}$$

or

$$\mathbf{S} = \frac{c}{4\pi} (\mathbf{E} \times \mathbf{H}) \text{ (cgs)} \tag{8.3a}$$

The source of radiation is characterized by the *radiant emittance* R_r, i.e., the total radiant flux emitted from a unit of surface area of the source.

Radiant illumination E_r measures the density of the radiant flux incident on a given surface. It is easy to see that with the same intensity of radiation the radiant illumination may be different depending on the orientation of the surface onto which the radiation falls. With a given intensity S the radiant illumination will be proportional to the sine of the angle between the direction of the flux and that of a normal to the surface onto which the flux is impinging.

The dimensions of all three quantities S, R_r and E_r coincide, namely,

$$[S] = [R_r] = [E_r] = MT^{-3} \tag{8.4}$$

The units in the SI and cgs systems are correspondingly W/m^2 and $erg/(s \cdot cm^2)$. In addition to the system units, the heat units $cal/(s \cdot cm^2)$, $kcal/(h \cdot m^2)$, etc., are used, as in measuring radiant flux.

The total quantity of radiant energy impinging during a certain time onto a unit of surface area is measured by the *radiant quantity of illumination* H_r, determined by the expression

$$H_r = \int_0^t E_r \, dt \tag{8.5}$$

Its dimension is

$$[H_r] = MT^{-2} \tag{8.6}$$

Besides radiant emittance, a source of radiation is characterized by the *radiant intensity* and the *radiance*. The radiant intensity I_r is defined as the radiant flux of a source per unit of solid angle in the given direction.For the same source, the radiant intensity may differ in different directions. The dimension of radiant intensity coincides with that of radiant flux, i.e., with that of power, since in the SI and cgs systems solid angle is a dimensionless quantity. The name of the units of radiant intensity contains the unit of solid angle steradian. The corresponding units are W/sr and erg/sr.

Radiance L_r is the radiant intensity per unit of area of the projection of the surface of the source onto a direction perpendicular to the direction of propagation of the radiation. According to this definition

$$I_r = \frac{dI_r}{dA'} \tag{8.7}$$

Here

$$dA' = dA \cos \alpha$$

where dA is the area of an element of the surface, and α is the angle between the direction of radiation and that of a normal to the surface.

If the radiation of a source of light complies with the Lambert law, according to which

$$I_r = I_{r0} \cos \alpha \tag{8.8}$$

where I_{r0} is the radiant intensity in a direction perpendicular to the surface of the source, then the radiance of the source is the same in all directions. Such sources are called *Lambert* ones.

The dimension of radiance is the same as that of radiant flux density:

$$[I_r] = MT^{-3} \tag{8.9}$$

Its units are $W/(m^2 \cdot sr)$ and $erg/(s \cdot cm^2 \cdot sr)$.

Radiant energy density u. The radiant energy per unit of volume is called the radiant energy density. The radiant energy density (see Sec. 4.4) is measured in J/m^3, erg/cm^3, etc.

The radiant energy density is of special interest if the radiation is concentrated in a closed space. Here the radiation obeys the laws of radiation of a black body, in particular the Stefan-Boltzmann law, according to which the volume density of radiation is proportional to the fourth power of the absolute temperature. If a small (in comparison with the total surface) hole is made in the shell containing the radiation, then this hole will be a black emitter whose radiant emittance is related to the radiant energy density by the expression

$$R_r = \frac{1}{4} uc \tag{8.10}$$

where c is the velocity of light in a vacuum. According to the Stefan-Boltzmann law

$$R_r = \sigma T^4 \tag{8.11}$$

where σ is a constant of this law,

$$\sigma = 5.669 \times 10^{-8}\ \mathrm{W/(m^2 \cdot {}^\circ K)} = 5.669 \times 10^{-5}\ \mathrm{erg/(s \cdot cm^2 \cdot {}^\circ K)}$$

Together with the radiant energy characteristics listed above that have an integral nature, that is, that do not relate to a definite range of the radiation spectrum, of great significance are spectral characteristics that are in essence functions of distribution of the given quantity by wavelength or by frequency.

Since the radiation of a source is not perfectly monochromatic, but is distributed in some manner along the spectrum, the action of the radiation may be quite diverse. In some instances we use the features of distribution of the given source, in others we convert the radiation of one spectral composition into that of another (for example, the conversion of ultraviolet radiation into visible light in luminescent lamps), and, finally, we sometimes have to ensure protection from a definite part of radiation, etc.

Since the concept of the function of distribution was considered in sufficient detail in Sec. 4.5, here we shall only give mathematical expressions for the corresponding spectral characteristics, the dimension formulas and units.

Spectral radiant flux density along the wavelength is determined as

$$\Phi_{r\lambda} = \frac{d\Phi_r}{d\lambda} \tag{8.12}$$

Its dimension is

$$[\Phi_{r\lambda}] = LMT^{-3} \tag{8.13}$$

Its units in the SI and cgs systems are respectively W/m and erg/(s·cm). In spectroscopy the flux is generally related to the interval of wavelengths measured in the units employed for the given region of the spectrum. For example, the units W/Å or erg/s·Å are used for the visible and adjacent regions of the spectrum.

Spectral radiant flux density along the frequency is expressed as

$$\Phi_{r\nu} = \frac{d\Phi_r}{d\nu} \tag{8.14}$$

Its dimension is

$$[\Phi_{r\nu}] = L^2MT^{-2} \tag{8.15}$$

In the following we shall not specially indicate whether the distribution is given by wavelength or frequency for spectral distributions, since this is obvious from the mathematical definition.

The spectral density of quantities determined by the radiant flux density (spectral density of energy flow, radiant emittance, radiant illumination) is equal to

$$S_\lambda = \frac{dS}{d\lambda}; \quad R_{r\lambda} = \frac{dR_r}{d\lambda}; \quad E_{r\lambda} = \frac{dE_r}{d\lambda} \tag{8.16}$$

$$S_\nu = \frac{dS}{d\nu}; \quad R_{r\nu} = \frac{dR_r}{d\nu}; \quad E_{r\nu} = \frac{dE_r}{d\nu} \tag{8.17}$$

$$[S_\lambda] = [R_{r\lambda}] = [E_{r\lambda}] = L^{-1}MT^{-3} \tag{8.18}$$

$$[S_\nu] = [R_{r\nu}] = [E_{r\nu}] = MT^{-2} \tag{8.19}$$

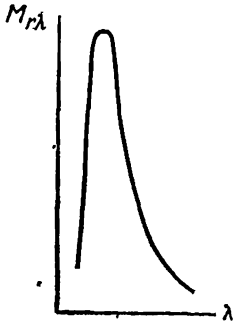

Fig. 24

The units of S, $R_{r\lambda}$ and $E_{r\lambda}$ are W/m^3 or erg/(s·cm^3), and those of S_ν, $R_{r\nu}$ and $E_{r\nu}$ are J/m^2 or erg/cm^2.

The spectral density of the radiant emittance of a black body, $M_{r\lambda}$, is shown in Fig. 24. The dimension of the spectral distribution of the radiant intensity coincides with that of the spectral distribution of the radiant flux. With respect to the relevant units, then in contrast to the units of $\Phi_{r\lambda}$ and $\Phi_{r\nu}$, their names show that they are related to a unit of solid angle, which is also reflected in the denominator of the symbols of these units.

In the same way the dimension of the spectral distribution of radiance coincides with that of the radiant flux density (i.e., energy flow, radiant emittance and radiant illumination), while the units are obtained from the corresponding ones by relating them to a unit of solid angle.

The dimension of the spectral distributions of the radiant energy density is

$$[u_\lambda] = L^{-2}MT^{-2} \tag{8.20}$$

$$[u_\nu] = L^{-1}MT^{-1} \tag{8.21}$$

It is general knowledge that for a black body u_λ and u_ν are determined by Planck's formula:

$$u_\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \tag{8.22}$$

$$u_\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \tag{8.23}$$

where h = Planck's constant
k = Boltzmann's constant
T = absolute temperature
e = base of natural logarithms.

The spectral distribution of the radiant emittance of a black body can be obtained from formulas (8.22) and (8.23) by multiplying them by $c/4$.

8.3. Illumination Engineering Units

Measurements of light have the feature that direct perception plays a very great part in them. Thus such measurements, strictly speaking, are not quite objective. Since in measurements of light we are interested only in the part of the total radiant flux that directly acts on our eye, the customary energy characteristics are not sufficient. Indeed, among the tremendous region of investigated electromagnetic oscillations only a narrow band of the visible spectrum with wavelengths ranging approximately from 0.38 to 0.76 micron are "optically valuable" for us or, as is said, have adequate visibility.

Various technical means are available that allow us to detect and measure radiation of electromagnetic waves of any range from long ones used in radio engineering to the shortest ones registered by counters of penetrating radiations, but, whatever the power of a radiation, we are "blind" with respect to it if its wavelengths are beyond the

limits of the interval indicated above. Moreover, even within this interval the sensitivity of our eye is different and, consequently, different regions of the visible spectrum have different visibility.

Practical illumination engineering poses many questions: what spectral composition of light should be considered as the most "natural" one, how can sources with a different spectral composition be compared, and many others. It is obviously essential to come to an agreement on some single methods to be used for comparing and measuring quantities that should characterize sources of light and the conditions of illumination.

It would seem most expedient to turn to natural sunlight, taking it as the prototype for purposes of comparison. It is easy to see that such a concept as natural sunlight is, however, very ambiguous. The time of the year, time of the day, geographical latitude, weather, altitude above sea level, purity of the atmosphere—all these factors change the quantitative and qualitative composition of sunlight in very broad limits. For this reason we have to come to an agreement on the selection of an artificial source of light that could be taken as an international prototype. Many such prototypes were proposed at various times (the Hefner candle, carcel, Viole standard, etc.), which at present have only a historical significance. The main drawback of these prototypes was the difficulty of reproducing them. It was obviously desirable to select such a source whose light radiation would be determined by physical laws as general as possible.

Since a black body is a universal radiator, its radiation was taken as the prototype. The temperature of the radiator must be fixed as accurately as possible, because radiation sharply grows with temperature. The temperature of solidification of platinum (2 042°K) was taken as such a temperature. The basic illumination engineering unit included among the basic units of the SI system is the unit of luminous intensity, the *candela* (cd), which is 1/60th of the radiation emitted by one square centimetre of a surface of a perfect radiator (black body) at the temperature of solidification of platinum. The international candle previously in use is equal to 1.005 cd.

The basic unit candela serves to determine all the other illumination engineering units. Since luminous intensity is included among the basic quantities, its symbol $\mathcal{J}$ appears in the dimension formulas.

Luminous flux is determined by the product of the luminous intensity and the solid angle formed by the flux. In the general case of non-uniform radiation

$$\Phi = \int_{\Omega} I \, d\Omega \tag{8.24}$$

With uniform radiation within the limits of the angle

$$\Phi = I\Omega \tag{8.24a}$$

With uniform radiation in all directions

$$\Phi = 4\pi I \tag{8.24b}$$

Before the introduction of the SI system the basic quantity in illumination engineering was the luminous flux, defined as the power of luminous radiation evaluated in terms of its visual effect. This definition underlines the subjective, physiological nature of illumination engineering quantities.

The unit of luminous flux is the *lumen* (lm)—the flux within a solid angle of one steradian with a luminous intensity of one candela. This unit is a basic one in the system of units of light based on the centimetre, gram and second, owing to which this system is generally designated cgsl. The dimension of luminous flux coincides with that of luminous intensity:

$$[\Phi] = \mathcal{J} \tag{8.25}$$

Quantity of light is the product of the luminous flux and the duration of its action

$$Q = \int_{t} \Phi \, dt \tag{8.26}$$

Its dimension is

$$[Q] = T\mathcal{J} \tag{8.27}$$

Its unit is the *lumen-second* (lm·s).

Similarly to the radiant energy quantities measured by the radiant flux density, the corresponding illumination engineering quantities and their units can be defined. Since these definitions are absolutely similar to those of their radiant energy counterparts, we shall limit ourselves to the formulas of the defining relationships and to the definitions of the units.

Luminous emittance

$$R=\frac{d\Phi}{dA} \tag{8.28}$$

The unit of luminous emittance is the luminous emittance of a source, each square metre of which produces a luminous flux of one lumen. This unit is called *lumen per square metre* ($\mathrm{lm/m^2}$). It was previously called the *radlux*. The unit in the cgsl system, the *lumen per square centimetre* ($\mathrm{lm/cm^2}$), is obviously 10^4 times greater than a $\mathrm{lm/m^2}$. This unit was previously called the *radphot*.

Luminous flux intensity, the same as luminous emittance, is measured in $\mathrm{lm/m^2}$ and $\mathrm{lm/cm^2}$.

Illumination

$$E=\frac{d\Phi}{dA} \tag{8.29}$$

The unit of illumination, the *lux* (lx), is the illumination of a surface each square metre of which receives a luminous flux of one lumen. In the cgsl system the relevant unit is the *phot*—the illumination of a surface, a square centimetre of which receives a flux of one lumen. Hence 1 lx $= 10^{-4}$ ph. Using the expression for the illumination of a surface by a source of light with an intensity of I candelas at a distance of r metres from the illuminated surface

$$E=\frac{I}{r^2}\cos\alpha \tag{8.30}$$

where α is the angle between the direction of the luminous flux and a normal to the illuminated surface, we can define the lux as the illumination of a surface at a distance of 1 metre from a source with a luminous intensity of 1 candela and arranged perpendicular to the incident light.

Luminance. The unit of luminance, the *nit* (nt), is the luminance of a source each square metre of whose radiating

surface has in the given direction a luminous intensity equal to one candela. The cgsl unit, the *stilb* (sb), equal to one candela per square centimetre, is 10^4 times greater than the nit.

Sometimes special units are employed to measure non-emitting surfaces. If a surface diffuses light perfectly in all directions, without absorbing it at all, such a surface has the properties of a Lambert source whose luminance, the same in all directions, is equal to

$$L = \frac{1}{\pi} E \tag{8.31}$$

The luminance of a perfectly white surface whose illumination is equal to one phot is called a *lambert* (L). It follows from formula (8.31) that 1 L $= \frac{1}{\pi}$ sb$=$0.318 sb. The luminance of the same surface with an illumination of one lux is sometimes called an *apostilb* (asb). Accordingly, 1 asb $= 10^{-4}$ L $= 3.18 \times 10^{-5}$ sb $= 0.318$ nt.

The units of all the quantities listed above (emittance, intensity, illumination, luminance) coincide in each of the systems, the only feature being that the dimension formulas in the SI system include the symbol of the dimension of luminous intensity ($\mathcal{J}$), and in the cgsl system—that of luminous flux (Φ). These dimensions are, respectively,

$$[R] = [E] = [L[= L^{-2}\mathcal{J} = L^{-2}\Phi \tag{8.32}$$

In addition to the quantities listed above, use is also made of *candela-second* — the product of luminous intensity and the duration of illumination

$$C = It \tag{8.33}$$

and the *quantity of illumination* — the product of illumination and its duration

$$H = Et \tag{8.34}$$

Candela-second, as its name implies, is measured in cd·s, and quantity of illumination in lx·s and ph·s.

8.4. Relationship between Subjective and Objective Characteristics of Light

The quantities whose units were considered in the previous two sections sharply differ from each other in the way of registering them. If the radiant energy quantities can be measured objectively with the aid of the relevant instruments, then in the final run the human eye is the main "instrument" by means of which illumination engineering quantities can be measured.

The question appears of how to bring into agreement subjective quantities appraised by arbitrary perception, and direct energy measurements. For this purpose it is obviously essential to take account only of the "valuable" part, and not the total energy radiated by a source of light, since any source, especially a heat one, radiates the predominate part of its energy beyond the limits of the visible part of the spectrum. Having selected a definite narrow part of the spectrum, we should measure the energy radiated in this part, and the luminous flux obtained with this energy. The task is complicated by the fact that the measurements have to be combined with subjective observations, and since the sensitivity to different colours varies appreciably in different people, the measurements have to be performed with the employment of a great number of observers so as to obtain sufficiently substantiated mean statistical values.

Investigations have shown that the "average eye" reacts differently to different regions of the spectrum. The sensitivity of the eye grows beginning from the shortest waves (about 0.4 μ), reaches its maximum at a wavelength of about 0.554 μ and then decreases again. This relationship is characterized by a special quantity named *luminous efficiency*.

By *absolute luminous efficiency* is meant the ratio of the luminous flux (i.e., the power appraised by our eye) to the corresponding true, total power of radiant energy

$$\eta = \frac{\Phi}{\Phi_r} \tag{8.35}$$

where η is the luminous efficiency, and Φ_r the power of radiant energy.

Customarily Φ_r is measured in watts, and since Φ is measured in lumens, then the unit of luminous efficiency

is lm/W. The ratio of the total luminous flux of white light to the corresponding radiant flux is generally called the *total luminous efficiency*, while the corresponding ratio for light having a definite wavelength is called the *monochromatic efficiency*.

Luminous efficiency is a special quantity that makes possible the conversion of radiant energy quantities into light

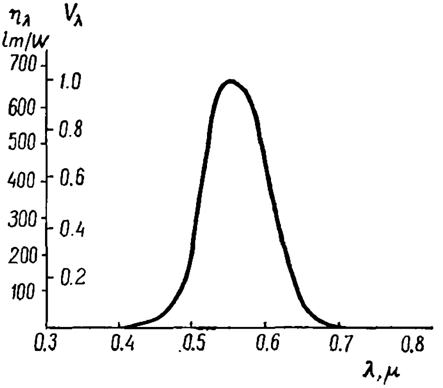

Fig. 25

ones. For this reason luminous efficiency is often selected as a basic quantity having its own dimension (η).

In this instance the dimension of luminous flux will be

$$[\Phi] = [\Phi_r][\eta] = L^2MT^{-3}\eta \qquad (8.36)$$

Relative luminous efficiency. As we have already noted, the luminous efficiency differs in different regions of the spectrum. The ratio of the luminous efficiency of a given wavelength to the maximum efficiency is called the relative luminous efficiency

$$V_\lambda = \frac{\eta_\lambda}{\eta_{max}} \qquad (8.37)$$

Figure 25 shows a curve of the spectral sensitivity of the eye, the wavelengths in microns being laid off along the axis of abscissas, and the relevant absolute and relative luminous efficiencies along the axis of ordinates. This curve

has been plotted using the data of Table 43. A glance at this table shows that the maximum luminous efficiency, at a wavelength of $\lambda = 0.554\ \mu$, lying in the green region of the spectrum, is 683 lm/W.

A source that would give up all its energy in the form of radiation only with a wavelength of $0.554\ \mu$ would be the most economical one. Such a source, however, would not suit us at all, because all the objects surrounding us would be coloured only green and would differ from one another only in that some would be brighter and others darker. It would be the best to have such a source that would radiate energy only in the visible region, and with such a distribution by wavelengths that would correspond to the conditional "mean sunlight". If we take as a unit the efficiency of a perfect source, i.e., such a source that radiates only light with a wavelength of $0.554\ \mu$, then the efficiency of a perfect "daylight" source would be 0.35. A source of heat radiation that is the closest to sunlight in the composition of its radiation is a black body at a temperature of about 6 000°K. Its efficiency is about 0.14. The efficiency of incandescent lamps is about 0.02, and that of luminescent lamps about 0.06.

Mechanical equivalent of light. As mentioned above, for the maximum luminous efficiency we have $\eta_{max} = 683$ lm/W. The reciprocal quantity of η_{max} is called the mechanical equivalent of light

$$M_l = \frac{1}{\eta_{max}} = 1.466 \times 10^{-3}\ \text{W/lm} \tag{8.38}$$

This is in essence the minimum mechanical equivalent of light, i.e., the minimum power in watts that is capable of creating a flux of one lumen in the region of the spectrum that is best perceived by the eye.

8.5. Units of Parameters of Optical Instruments

In this section we shall deal with the units of quantities characterizing the optical properties of instruments. In essence they should be related to the group of geometrical quantities, but since they are encountered in optics, we

have found it more expedient to include them in the chapter relating to the theory of radiation.

Lens power. If a plane wave (parallel rays) impinges on the lens of an instrument and the lens imparts to it a curvature with a radius f, then we say that the lens has a power of

$$D=\frac{1}{f} \tag{8.39}$$

The unit of lens power is the power of such a lens that imparts to a plane wave a curvature with a radius of one metre.

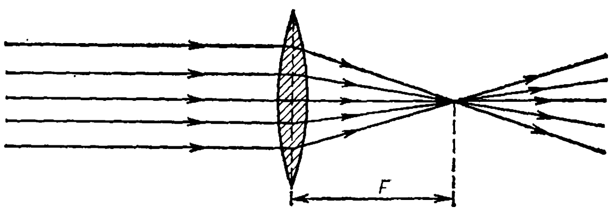

Fig. 26

This unit is called the *dioptre*, and depending on the direction of the radius of curvature, is considered to be either positive (converging rays) or negative (diverging rays).

The dimension of the dioptre is

$$[D]=L^{-1} \tag{8.40}$$

Principal focal length. The quantity f, which is the reciprocal of lens power, is called the principal focal length of a lens and is usually measured in metres or centimetres.

With a thin lens the principal focal length is the distance from the lens to the principal focus, i.e., to the point in which the rays gather that impinge on the lens parallel to its principal optical axis (Fig. 26).

The principal focus of a diverging lens is the point at which the continuations of the diverging rays obtained when a beam of parallel rays impinges on the lens will intersect (Fig. 27).

For a complicated centred optical system the principal focal length is measured from the principal focus, i.e., the point of real or virtual intersection of the rays leaving

the system when they enter it parallel to the principal optical axis, to the principal plane—the plane in which

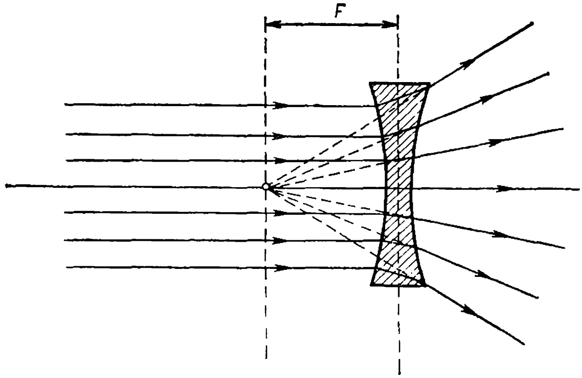

Fig. 27

the directions of the incident and emergent rays intersect (Fig. 28).

Relative aperture (*f number*) is the ratio of the principal focal length of a lens to the diameter of its entrance pupil

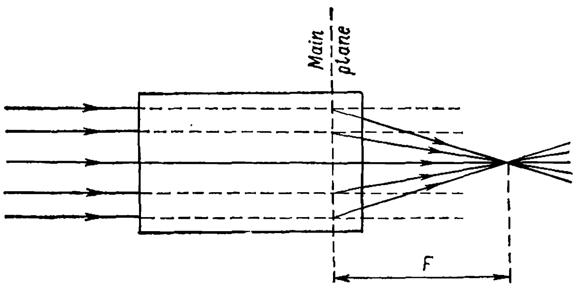

Fig. 28

(aperture). It is generally written as a fraction with *f* in the numerator and the ratio in the denominator. Thus, it is said that a camera has a relative aperture (or *f* number) of *f*/2.8. This quantity is an abstract dimensionless one.

8.6. Units of Optical Properties of a Substance

Refractive index n. Upon the refraction of light at the interface between two isotropic media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction (the angles are measured from a perpendicular to the interface) remains constant. This ratio is called the refractive index of the second medium with respect to the first one, or the relative refractive index.

When light enters a given medium from a vacuum, we call this ratio the absolute refractive index, thus assuming the refractive index of a vacuum to be equal to unity. It follows from the definition that the refractive index is an abstract (dimensionless) quantity.

Absorption factor is the ratio of the energy absorbed by a body to the total energy impinging on it. This factor is also a dimensionless quantity.

Light factors. When a luminous flux impinges on the surface of a body, part of the flux is directly reflected (according to the law of reflection), part is more or less uniformly diffused in all directions, part is absorbed, and part is transmitted through the body. The ratios of these luminous fluxes to the total incident flux are respectively called the *reflection factor* ρ, the *diffusion factor* σ, the *absorption factor* α, and the *transmission factor* τ (*transparency*). It is good practice to relate the latter two factors to a unit of layer thickness. In particular, the absorption factor related to a unit of length can be determined from the formula

$$I = I_0 e^{-\alpha x} \tag{8.41}$$

where I_0 is the intensity of the incident light, and I the intensity of the light passing through the thickness x.

The dimension of the factor α is

$$[\alpha] = L^{-1} \tag{8.42}$$

since the exponent must be dimensionless.

CHAPTER NINE

SELECTED UNITS OF ATOMIC PHYSICS

9.1. Introduction

The progress of atomic physics gave birth to a great number of specific methods for measuring the properties of atomic particles, the quantities characterizing the processes which they participate in, etc. It was often found convenient to introduce special units, partly based on units of the cgs system, partly of a mixed nature, and sometimes not directly related to any definite system.

Lately a tendency has appeared in literature on the subject to use units of one of the general systems (most frequently the SI) for all such measurements, but the predominant majority of scientific articles retain the units that were always used in atomic physics. Without undertaking the task of dealing with all these units, we shall consider the most important and widely used of them.

9.2. Basic Properties of Atomic and Elementary Particles

Mass. The mass of particles can be measured either absolutely or relatively. By absolute measurement we mean the use of one of the generally adopted units of mass (kg, g) and by relative measurement—comparison of a given mass with that of a particle conventionally accepted as a unit. Such a unit is the *atomic mass unit* (amu), which has undergone certain changes during a number of years. Previously in chemistry the atomic mass unit was taken as one-sixteenth of the atomic weight of oxygen in its natural state, and

in physics as one-sixteenth of the mass of the lightest isotope of oxygen whose mass number is sixteen. It should be remembered that the mass number is an integer equal to the total number of nucleons (i.e., protons and neutrons) in the nucleus.

Since natural oxygen contains three stable isotopes with mass numbers of 16, 17 and 18, and whose content is 99.76%, 0.04% and 0.20% respectively, then the atomic mass unit used by chemists was 1.000272 times greater than that used by physicists.

The use of the physical atomic mass unit defined above had a number of incoveniences due to the fact that the precise determination of atomic masses was experimentally related not to atoms of oxygen, but to atoms of carbon. For this reason in 1961 the International Union of Pure and Applied Physics and the International Union of Pure and Applied Chemistry decided to establish the atomic mass unit (both in physics and chemistry) as one-twelfth of the mass of the carbon isotope with a mass number of 12 (C_6^{12}). This unit is equal to 1.0003179 old "oxygen" physical units. It is very close to the old chemical mass unit, differing from the latter only by several units in the fifth decimal place.

The atomic mass unit is equal to 1.6604×10^{-27} kg. The atomic weights of elements, molecular weights (relative molecular masses) of chemical substances and the masses of nuclei are determined relative to the atomic mass unit. The masses of elementary particles are generally related to the mass of an electron m_e, equal to 9.109×10^{-31} kg or 5.486×10^{-4} amu.

Charge. Atomic and elementary particles are either deprived of a charge, or have a positive or negative charge which is a multiple of that of an electron. The latter is equal to 1.6021×10^{-19} C $= 4.803 \times 10^{-10}$ cgs_Q.

Moment of momentum (*angular momentum*) of microparticles obeys the laws of quantum mechanics, according to which it can have only definite discrete values, determined by the expression

$$L = \frac{h}{2\pi}\sqrt{j(j+1)} \tag{9.1}$$

where h is the Planck constant, and j the quantum number of the moment of momentum. The Planck constant $h = 6.626 \times 10^{-34}$ J·s $= 6.626 \times 10^{-27}$ erg·s.

The ratio $\frac{h}{2\pi} = 1.0545 \times 10^{-34}$ J·s $= 1.0545 \times 10^{-27}$ erg·s that is generally used in quantum mechanics instead of h is designated $\hbar$. The quantum number j may be an integer, a half-integer $\left(\text{an odd multiple of } \frac{1}{2}\right)$, or zero. For an electron the quantum number of the moment of momentum is designated $s = \frac{1}{2}$ and is called the *spin number*. Hence the own moment of momentum of an electron is

$$L = \frac{h}{2\pi}\sqrt{\frac{3}{4}} = 0.913 \times 10^{-34} \text{ J·s} = 0.913 \times 10^{-27} \text{ erg·s} \tag{9.2}$$

The quantity

$$\hbar = \frac{h}{2\pi} \tag{9.3}$$

serves in atomic physics as the unit of the moment of momentum (angular momentum).

Magnetic moment. In the classical Bohr theory an electron revolving in a circular orbit about a nucleus forms a closed current that, consequently, has its own magnetic moment. Quantum mechanics, while renouncing the illustrative model representations (the "orbit" of an electron in an atom, the "revolving electron"), retains such quantities as the moment of momentum considered above and, correspondingly, the magnetic moment.

In the classical model, the magnetic moment of an atom of hydrogen in the normal (unexcited) state is easily calculated as follows. The ratio of the charge of an electron to the period of its revolution in an atom is the "current intensity"

$$I = \frac{e}{T} = \frac{e}{2\pi}\omega \tag{9.4}$$

According to Bohr's postulate

$$m\omega a_0^2 = \frac{h}{2\pi} \tag{9.5}$$

where a_0 is the radius of the orbit (the so-called Bohr radius). Consequently

$$I = \frac{eh}{4\pi^2 m a_0^2}$$

and the magnetic moment (designated here μ_B) is

$$\left.\begin{aligned} \mu_B &= \frac{eh}{4\pi m}\ (\mathrm{SI}) \\ \mu_B &= \frac{eh}{4\pi mc}\ (\mathrm{cgs}) \end{aligned}\right\} \qquad (9.6)$$

The magnetic moment determined by formula (9.6) is called the *Bohr magneton* and serves as a unit of magnetic moment. Its value is

$$\mu_B = 9.273 \times 10^{-24}\ \mathrm{A\cdot m^2} = 9.273 \times 10^{-21}\ \mathrm{dyn\cdot cm/Gs} \qquad (9.7)$$

The magnetic moments of nuclear particles are measured with the aid of the so-called *nuclear magneton* (μ_N), which is determined by the same formula (9.6), but with the mass of a proton substituted for that of an electron, the former being 1 836 times greater than the latter. Hence the nuclear magneton is

$$\mu_N = 5.051 \times 10^{-27}\ \mathrm{A\cdot m^2} = 5.051 \times 10^{-24}\ \mathrm{dyn\cdot cm/Gs} \quad (9.8)$$

It should be noted here that the magnetic moments of nuclei are not integer multiples of the nuclear magneton, but are calculated by a quite complicated formula. In particular, the magnetic moment of a proton is

$$\mu_p = 2.7928\mu_N = 1.4105 \times 10^{-26}\ \mathrm{A\cdot m^2} = \\ = 1.4105 \times 10^{-23}\ \mathrm{dyn\cdot cm/Gs}$$

Dipole moment. Polarization. The electric charges in molecules may be distributed unsymmetrically, as a result of which the molecule as a whole acquires an electric dipole moment. The dipole moment is measured either in cgs or SI units (see Secs. 7.3 and 7.4), or in a special unit, the *debye* (D), equal to 10^{-18} cgs units or 3.336×10^{-30} C·m.

Atoms and molecules, even when not having their own dipole moment, can acquire it under the action of an external field as a result of electronic polarization.

The ratio of the acquired dipole moment to the field intensity is called *polarization* α. According to the definition

$$\alpha = \frac{\vec{p}}{E} \tag{9.9}$$

The dimension of α in the cgs system is

$$[\alpha] = L^3 \tag{9.10}$$

and its name is cubic centimetre.

In the SI system the relevant dimension is

$$[\alpha] = M^{-1}T^4I^2 \tag{9.11}$$

From the definition of α according to formula (9.9), upon inserting $\vec{p} = 1$ C·m and $E = 1$ V/m and making the corresponding substitutions, it is easy to find that the SI unit of polarization is 9×10^{15} times greater than the cgs unit. The same relationship can be found from the dimension formulas. Polarization is related to relative permittivity (if the latter is determined only by electronic polarization) by the expression

$$\alpha = \frac{\varepsilon_r - 1}{N} \tag{9.12}$$

where N is the concentration of molecules of the given substance.

Lifetimes. Many elementary and atomic particles are not stable and after a certain time either disintegrate, or pass over into another state. To characterize the stability of atomic radioactive nuclei, the concept *half-life* is used. This is the time during which half the initial number of atoms disintegrate. Since the change in the number of radioactive atoms follows the exponential law

$$N = N_0 e^{-\lambda t} \tag{9.13}$$

where N_0 is the initial number of atoms, N the number of undecayed atoms after the time t has elapsed, and λ is the so-called *decay* or *disintegration constant*, then the half-life will be determined by the expression

$$\frac{N_0}{2} = N_0 e^{-\lambda t} \tag{9.14}$$

whence

$$T = \frac{\log_e^2}{\lambda} = \frac{0.693}{\lambda} \qquad (9.15)$$

To characterize the stability of atoms in an excited state, the concept of *mean lifetime* τ is used, found from the exponential law

$$N = N_0 e^{-t/\tau} \qquad (9.16)$$

The mean lifetime is equal to the time during which the number of atoms in an excited state will be reduced to $1/e$ of the original number. The half-life and the mean lifetime are obviously related by the following expression

$$T = 0.693\tau \qquad (9.17)$$

Linear dimensions. In quantum mechanics such concepts as "the radius of an electron orbit", the radius of an elementary particle (for example, of an electron), etc., have no meaning. It is often convenient, however, to introduce definite linear scales whose capacity is filled by quantities obtained on the basis of classical calculations. The most widespread of these are the "classical electron radius" determined by the expression

$$r_0 = \frac{e^2}{mc^2} = 2.818 \times 10^{-13} \text{ cm} \qquad (9.18)$$

and "the radius of the first Bohr orbit"

$$a_0 = \frac{\hbar}{me^2} = 0.5292 \times 10^{-8} \text{ cm} \qquad (9.19)$$

In addition, in nuclear physics use is made of a unit of length called the *fermi*, equal to 10^{-13} cm.

9.3. Effective Interaction Cross Sections

The classical kinetic theory of gases introduced the concept of the free path, relating it to the concept of the cross section of colliding particles. Atomic physics has extended the concept of cross section and has simultaneously divided it, with the establishment of the concept of effective cross section relative to a concrete process of interaction of atoms,

ions, molecules, nuclear particles, etc. The concept of *effective cross section* with respect to a process will be best explained using the following semiclassical diagram, which we shall consider with respect to a concrete example of the excitation of an atom by an electron impact (Fig. 29). Let an electron having a given velocity move perpendicular to the plane of the drawing toward the atom with an impact parameter r. By impact parameter is meant the length of a perpendicular erected from the centre of the atom to the vector of the initial velocity of the electron. Let, further, the probability of excitation of the atom with the given impact parameter be $p(r)$. Let us depict a ring confined within the radii r and $r + dr$ and separate on it a part equal to

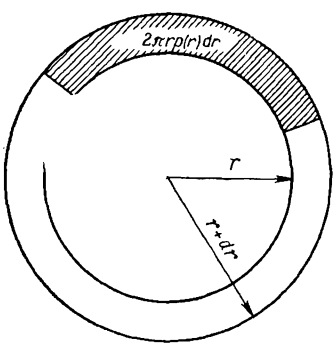

Fig. 29

$$d\sigma = p(r)\, 2\pi r\, dr \tag{9.20}$$

The quantity that we get by integration of $d\sigma$ with respect to all the values of r from 0 to ∞, namely,

$$\sigma = 2\pi \int_0^\infty p(r)\, r\, dr \tag{9.21}$$

is called the *effective cross section of excitation* of an atom by an electron having a given velocity. That σ has the dimension of area can be seen from the defining expression. With respect to the physical meaning of σ, it can be easily seen from the definition that the effective cross section is the section an atom must have for excitation to take place with a probability of 100% upon each impact of an electron.

The concept of effective cross section is exceedingly widely used in atomic and nuclear physics and in the fields of physics investigating macroscopic processes connected with the interaction of atomic particles. It is employed

to describe quantitatively all kinds of elastic and inelastic processes of interaction.

Various units are used to measure effective cross sections. In nuclear physics the corresponding unit is the *barn* (b), 1 b = 10^{-28} m^2 = 10^{-24} cm^2. In the physics of atomic collisions use is made of the cm^2, less frequently the m^2, and the units a_0^2 and πa_0^2 (where a_0 is the radius of the first Bohr orbit):

$$a_0^2 = 0.2800 \times 10^{-16}\ \text{cm}^2$$
$$\pi a_0^2 = 0.8797 \times 10^{-16}\ \text{cm}^2$$

In scientific literature the term *reduced effective cross section* is sometimes used. This is the sum of the corresponding effective cross sections of all the atoms or molecules contained in 1 cm^3 at 0°C and 1 mm Hg. Since in these conditions the number of molecules in 1 cm^3 is 3.54×10^{16}, then we shall obtain the reduced effective cross section by multiplying by this number the effective cross section measured in $cm^2/(cm^3 \cdot mm\ Hg)$. The unit of reduced effective cross section is usually designated by $cm^2/(cm^3 \cdot mm\ Hg)$.

9.4. Units of Energy in Atomic Physics

Electron-volt. If an electron travels through a potential difference without losing any energy on its way, it acquires the kinetic energy

$$\frac{m_e v^2}{2} = eU \tag{9.22}$$

(assuming that $v_0 = 0$).

When the potential difference is 1 V, the energy will be

$$1\ \text{eV} = 1.60207 \times 10^{-19}\ \text{B} \times 1\ \text{V} = 1.60207 \times 10^{-19}\ \text{J}$$
$$= 1.60207 \times 10^{-12}\ \text{erg} \tag{9.23}$$

It is very often convenient to use this energy as a unit for measuring not only the energy of electrons, but also that of other particles or energy levels in atoms and molecules. This unit of energy is called the *electron-volt* (eV). Units that are 10^3, 10^6 and 10^9 times greater are frequently used—keV, MeV, and GeV.

The velocity acquired by an electron after travelling through a potential difference of U volts can also be determined by formula (9.22)

$$v=\sqrt{\frac{2eU}{m}}=5.932\times 10^5\sqrt{U}\ \mathrm{m/s} \qquad (9.24)$$

Thus, the velocity of an electron uniquely determines the potential difference it acquires. This is why it is often said that "an electron has a velocity of U volts", meaning that it has such a velocity which it would acquire if it travelled through a potential difference of U volts. The velocity of an electron in volts is converted to velocity in m/s or cm/s by means of formula (9.24).

Relationship between electron-volt and degree Kelvin. If a gas is at a temperature T, then the mean kinetic energy of translational motion of its molecules is equal to

$$\frac{mv^2}{2}=\frac{3}{2}kT \qquad (9.25)$$

If an elementary charge were connected with each molecule, then the molecules could acquire their energy by travelling through a potential difference U determined by the relationship

$$\frac{3}{2}kT=eU \qquad (9.26)$$

If $U=1$ V, then the corresponding temperature is

$$T_{(1\mathrm{V})}=\frac{2\times 1\ \mathrm{eV}}{3\times k}=\frac{2\times 1.602\times 10^{-19}}{3\times 1.380\times 10^{-23}}=7\,733^\circ\mathrm{K} \qquad (9.27)$$

Many equations of statistical physics, thermodynamics, spectroscopy, etc., include the exponent $e^{E/kT}$, where E is the energy of transition from one state to another. When measuring this energy in electron-volts, it will be convenient to write the denominator in electron-volts too. The latter will be equal to one electron-volt if $T\cong 11\,600^\circ$K.

Relationship between electron-volt and calorie per mole. If each of the molecules contained in one mole acquires an energy of one electron-volt, then the total energy of all the molecules will increase by

$$1\ \mathrm{eV}\times N_A=1.602\times 10^{-19}\times 6.023\times 10^{23}\ \mathrm{J}=9.749\times 10^4\ \mathrm{J} = 23\,053\ \mathrm{cal} \qquad (9.28)$$

Relationship between energy measured in electron-volts and length of a light wave. Each spectrum line is characterized by a definite wavelength or frequency and, consequently, a definite quantum of energy

$$h\nu = \frac{hc}{\lambda} \tag{9.29}$$

For this reason it is possible to establish the relationship between the wavelength of a given spectrum line measured in angstroms and the energy corresponding to it, measured in electron-volts. The importance of this grows because of the fact that an atom is frequently excited to a higher energy level by an electron impact, upon the transition from which to the normal state it radiates a quantum of energy.

From the relationship

$$eU = h\nu = \frac{hc}{\lambda} \tag{9.30}$$

it is easy to obtain

$$\lambda U = (12\,395 \pm 2)\ \text{Å V} \tag{9.30a}$$

Relationship between velocity of electron measured in electron-volts and the length of its De Broglie wave. The following wavelength is connected with an electron moving with a velocity v:

$$\lambda = \frac{h}{m_e v} \tag{9.31}$$

By expressing the velocity of the electron in electron-volts (see Sec. 9.24) and substituting the corresponding values for h and m_e, we get

$$\lambda = \frac{12.26}{\sqrt{U}} \tag{9.31a}$$

Here U is measured in volts, and λ in angstroms. It is convenient to use the approximate expression

$$\lambda \cong \sqrt{\frac{150}{U}} \tag{9.31b}$$

a glance at which shows that an electron with an energy of 150 eV has a De Broglie wavelength of 1 Å.

Relationship between mass and energy. The theory of relativity gives the following relationship between mass and energy:

$$E = mc^2 \tag{9.32}$$

It is known that the measured difference between the mass of an atomic nucleus and the sum of the masses of the protons and neutrons forming it can be used to compute the nucleon binding energy in a nucleus. Below are given the most favoured approximate relationships between the units of mass and energy.

For macroscopic bodies

$$1\ \text{kg} \equiv 9 \times 10^{16}\ \text{J}$$

$$1\ \text{g} \equiv 9 \times 10^{20}\ \text{erg}$$

The energy corresponding to one atomic mass unit is

$$1\ \text{amu} \equiv 1.493 \times 10^{-10}\ \text{J} = 931.7\ \text{MeV} \tag{9.33}$$

For elementary particles

$$1\ m_e \equiv 8.18 \times 10^{-14}\ \text{J} = 0.511\ \text{MeV}$$

$$1\ m_p \equiv 1.50 \times 10^{-10}\ \text{J} = 938\ \text{MeV}$$

Unit of energy rydberg. The spectrum lines of atoms similar to hydrogen are arranged in a series satisfying the formula

$$\tilde{\sigma} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \tag{9.34}$$

where $\tilde{\sigma}$ = wave number of the given line

n_1 and n_2 = quantum number of the energy levels between which the transition is accompanied by the radiation of the corresponding quantum

R = so-called Rydberg constant.

For an infinitely massive atomic nucleus

$$R_\infty = 109\,737.3\ \text{cm}^{-1}$$

For light atoms the values of Rydberg's constant are somewhat smaller. For example, for a hydrogen atom

$$R_H = 109\,677\ \text{cm}^{-1} \tag{9,34a}$$

Multiplying both parts of equation (9.34) by hc, we get the value of the energy of an emitted quantum. The product in the right-hand part of the equation

$$R_y = Rch \tag{9.35}$$

is called the *rydberg* (R_y) and is used as a measure of the energy of electron levels. Assuming that $n_1 = 1$ and $n_2 = \infty$ we can define the rydberg as the energy that would have to be spent to ionize an atom of hydrogen if its mass were equal to infinity. Substitution of values for R, c and h yields

$$1\ R_y = 13.60\ \text{eV} \tag{9.36}$$

The actual energy of ionization of a hydrogen atom computed using the value of R_H is equal to 13.57 eV.

9.5. Ionizing Radiation Units

Atomic particles (electrons, atoms, ions, nuclear particles) and photons with a sufficiently great energy are capable of causing ionization of a gas when absorbed in it. This ability determines the experimental methods of registering such radiation and its quantitative characteristics. This is why in addition to radiant energy units determining the power of radiation, the radiant flux, etc., use is also made of certain specific units, in particular such that are employed to measure the ability of the given radiation to produce a definite ionization of a gas. Some of these units have been constructed on the basis of SI units, and others on the basis of cgs ones.

Particle or quantum flux is the number of particles or quanta passing through a given surface in a unit of time. The flux is generally related to a second and is correspondingly determined by the number of particles (α, β, etc.) or quanta per second. The particle flux passing through a unit of area perpendicular to its direction is called the flux density. Depending on the system of units used for the measurement, the flux is related to one square metre or centimetre. The flux and flux density may be measured not by the number of particles, but by the energy transferred (watts or ergs per second or, correspondingly, watts per square metre or ergs per second per square centimetre). In

this case they are respectively called the radiant flux and radiant flux density. Both these quantities and their units completely coincide with the radiant energy characteristics considered in Sec. 8.2.

We have already considered the radiant energy density and its units. In *investigation of ionizing radiation* this quantity acquires a special importance, since it characterizes the energy absorbed in a unit of volume as a result of the transition of radiant energy into other forms. The total quantity of absorbed energy is measured in the common energy units (J, erg), while the density of absorbed radiant energy is measured in the same units as radiant energy density (J/m^3, erg/cm^3). In measuring the ionization caused by radiation, however, a more important quantity is the relation of the absorbed energy not to the volume, but to the mass of the absorbing substance. This is easy to understand if we consider absorption in a gas. Since ionization occurs upon the interaction of the emitted particles or quanta with atoms or molecules of the gas, then it is obvious that when the pressure is halved, the volume of the gas must be doubled to obtain the same ionization. The energy absorbed by a unit of mass of a given substance is called the *absorbed radiation dose* D. Its dimension is

$$[D] = L^2T^{-2} \tag{9.37}$$

and J/kg and erg/g are its units, 1 J/kg = 10^4 erg/g. The *rad*, a unit equal to 10^{-2} J/kg or 10^2 erg/g is also used. The absorbed radiation dose related to the duration of absorption is called the *power of absorbed radiation dose* p. Its dimension is

$$[p] = L^2T^{-3} \tag{9.38}$$

and its units are W/kg and erg/(s·kg).

To describe radiation from the viewpoint of the ionization it produces, a quantity called *radiation exposure* is used. In the SI system the corresponding unit is the *coulomb per kilogram* (C/kg)—an exposure producing in one kilogram of dry air a number of ions whose total charge is one coulomb of each sign. The cgs unit, the *roentgen* (r), is defined as the exposure to X rays or gamma rays upon which, as a result of complete ionizing absorption, ions with a tota

charge of 1 cgs unit of each sign are formed in one cubic centimetre of air in standard conditions. This corresponds to 2.082×10^9 pairs of ions in 1 cm^3 or 1.6×10^{12} pairs of ions in one gram of air.

Since the mass of a cubic centimetre of air at 0°C and 760 mm Hg is equal to 1.293×10^{-6} kg, then, with a view to the relationship between the coulomb and the cgs unit of charge, we get $1\ r = 2.58 \times 10^{-4}$ C/kg and 1 C/kg = $= 3.88 \times 10^3$ r. In practical work use is generally made of the submultiple units *microcoulomb per kilogram* (μC/kg), *milliroentgen* (mr), and smaller ones.

The units of radiation exposure serve to form the units of *dose rate*—coulomb per kilogram per second and roentgen per second.

The measurement of a radiation dose according to its ionization ability makes it possible to establish a physical equivalent of the unit of dose. Taking into account the mean energy spent for the ionization of a molecule of air (about 33 eV), it can be computed that 1 r is equivalent to 85 erg/g. This quantity is called the *physical roentgen equivalent* (designated rep). In appraising the biological action of radiation the *man roentgen equivalent*, designated rem, is employed.

The *rad*, equal to 100 erg/g, is used together with the roentgen to measure the total energy absorbed by a unit of mass of a substance.

A special unit, the *einstein* (E), sometimes used in the investigation of photochemical processes, can be included to a certain extent among the units of ionizing radiation. This unit is defined as $N_A\, h\nu$, where N_A is Avogadro's number and $h\nu$ is the energy of a quantum. It follows from this definition that the einstein unit is not a general unit of energy, but has different values depending on the wavelength of light.

9.6. Units of Radioactivity

The main process that is to be measured and registered in radioactive conversions is *disintegration* (*decay*) accompanied by the emission of alpha or beta particles, neutrons and gamma quanta. For this reason the unit characterizing

the activity of a radioactive source is the *disintegration per second* (d/s). The *rutherford* (Rd) is a unit of radioactivity equal to 10^6 d/s.

In addition to the units disintegration per second and rutherford, a unit of radioactivity the *curie* (c), now defined as the quantity of any radioactive isotope in which the number of disintegrations per second is 3.7×10^{10}, is also in use, although fractions of this unit are in greater favour. Previously the curie was defined as the radioactivity associated with the quantity of radon in equilibrium with one gram of radium. This can be explained as follows.

If radium is placed in a closed vessel, then initially the quantity of radon (radium emanation), which is a product of radium disintegration, will grow, but since radon itself also decays (with a half-life of 3.82 days), then equilibrium will finally set in between the newly appearing radon and the decaying one. The number of disintegrations per second will remain practically constant, if no consideration is taken of the change in the mass of the radium itself, which occurs very slowly, with a half-life of about 1 600 years. For this reason the radioactivity of radium can be compared with that of radon in equilibrium with a certain amount of radium. This was the origin of the previous definition of the curie. The quantity of radon corresponding to a radioactivity of 1 curie has a mass of 6.51×10^{-6} g and contains 1.78×10^{16} atoms. The alpha particles emitted by radon (not counting the products of its further decay) are capable of creating an ionization saturation current of 0.92 mA in air.

To measure the concentration of a radioactive preparation, use is sometimes made of the unit *eman*, a concentration of 10^{-10} curies per litre of fluid.

Formerly a unit of concentration, the *mache*, was used, which is equal to 3.46 emans, and is defined as the quantity of radioactive emanation (alpha-particles) of radon that sets up an ionization saturation current of 10^{-3} cgs units of current in air. This unit is now obsolete.

The particles emitted by a radioactive preparation form a flux measured by the number of particles per second. The number of particles per unit of surface area (square metre or square centimetre) is the particle flux density.

9.7. Ionization, Recombination and Mobility Coefficients

The properties of electrons, ions, atoms and other particles are described by various quantities inherent in the given particles and characterizing separate acts of interaction of these particles with one another, with quanta of radiation, etc. These quantities include, in particular, the effective cross sections considered above. To describe phenomena in which a great number of particles are involved, however, it is often convenient to use mean macroscopic quantities. This is encountered, for example, in the kinetic theory of gases when describing transfer phenomena (diffusion, viscosity, heat conductivity), i.e., phenomena characterized by macroscopic coefficients whose values can be found with the aid of the molecular theory.

In the present section we shall give several such quantities and their units as applied to the motion of charged particles in a gas.

Volume electronic ionization coefficient. When moving in an electric field, an electron acquires the ability of ionizing a gas. The mean number of ionizations N_i performed by an electron on a unit of its path in the direction of the field is called the *volume electronic ionization coefficient* or the *first Townsend coefficient*. The latter name is due to the fact that this coefficient was introduced by F. Townsend in his theory of semi-self-maintained discharge in a gas. The number N_i is measured in units that are reciprocals of the unit of length (m^{-1}, cm^{-1}).

Similar coefficients with the same units can be introduced for describing ionization by other particles (for example, by ions).

Recombination coefficient. If a gas contains charged particles of both signs with concentrations of n_+ and n_-, then the process of recombination of these particles into neutral atoms or molecules may occur. The number of such recombinations occurring in a unit of time in a unit of volume is determined by the equation

$$N_r = An_+n_- \tag{9.39}$$

where A is the recombination coefficient. Its dimension

$$[A] = L^3T^{-1} \tag{9.40}$$

determines its units, namely, m³/s and cm³/s.

Mobility coefficient (*mobility*). The velocity of a charged particle moving in a certain medium in an electric field will be established at a definite mean level owing to numerous collisions. There are distinguished the chaotic undirected velocity and the directed or drift velocity along the direction of the field. The latter determines the passage of an electric current. In the general case the directed (drift) velocity u may depend in a complicated way on the field intensity. In definite conditions the directed velocity will be proportional to the field intensity E:

$$u = bE \tag{9.41}$$

where b is the mobility coefficient or, as it is called more frequently, the mobility of a given charged particle.

Einstein showed that the mobility is related to the diffusivity D by the equation

$$\frac{D}{b} = \frac{kT}{e} \tag{9.42}$$

where k = Boltzmann's constant
e = charge of an electron
T = absolute temperature.

Equation (9.31), in particular, is complied with upon the motion of electrons in a metal, thus ensuring the applicability of Ohm's law to metals. Mobility can be defined as the mean directed velocity acquired by a particle when moving in a field whose intensity is unity. The dimension of mobility in the SI system is

$$[b] = LM^{-1}T^2I \tag{9.43}$$

and in the cgs system

$$[b] = L^{3/2}M^{-1/2} \tag{9.43a}$$

In practical work mobility is measured in the units $m^2/V \cdot s$ and $cm^2/V \cdot s$.

With other conditions equal, mobility, the same as diffusivity, is inversely proportional to the density of a gas or

the reduced pressure. This is why the concept of reduced mobility is frequently used, defined by the expression

$$b_1 = \frac{b}{p} \tag{9.44}$$

The reduced mobility is usually related either to 1 mm Hg or to a standard atmosphere.

9.8. Natural Systems of Units

It has been indicated more than once in this book that a single-valued relationship exists between the number of basic units and the number of universal constants, i.e., the greater the number of basic units, the more constants there are in the formulas of physical laws and definitions. By equating the gravitational constant to unity while simultaneously retaining the inertial constant equal to unity, we reduced the number of basic units in systems of geometrical and mechanical units from three to two. By equating Boltzmann's constant to unity, we make the unit of temperature a derived one. In systems of electrical and magnetic units we can further reduce the number of basic units by equating to unity the electric and magnetic constants in a system constructed according to the principle of the SI, or the velocity of light in a system constructed according to the principle of the cgs system. Thus we remain with two units, one of which, light intensity, reflects the specific physiological nature of perception of light, while the other may be either the unit of length or that of time, as we wish.

To what extent have we used all the possibilities of reducing the number of universal constants? Although the total number of such constants is comparatively great, it can be proved, however, by analysing the origin of the relevant equations, that as a result of the reduction of the number of units made above, almost all the constants will become equal either to unity or to a dimensionless constant number obtained as the result of a mathematical operation.

Nevertheless, even after the reduction of the number of basic units of all the quantities to one (leaving aside the unit of luminous intensity as not directly related to the

common physical quantities), we shall still have "unused" constants. These are the Planck constant h and the charge of an electron e. It is easy to see that in a system with one basic unit—length—the dimensions of these constants will be

$$[h] = L^2 \tag{9.45}$$

$$[e] = L \tag{9.46}$$

It is possible to deal with another constant, for example, h, and so select the unit of length that the Planck constant will be equal to unity. Here we shall obtain a system that is in general dimensionless, i.e., such a system in which we are completely deprived of freedom in selecting the dimension of any unit whatsoever.

Instead of equating the Planck constant to unity, we can do this with the charge of an electron; thus the Planck constant will be uniquely determined. We can, finally, equate to unity both the Planck constant and the charge of an electron, but in this instance a different constant will appear, the velocity of light, that will now differ from unity.

Systems in which the greatest possible number of universal constants have been equated to unity are called *natural systems*. The system described above in which $h = 1$ was proposed by Max Planck. In this system the unit of length is found to be 4.02×10^{-33} cm, of mass $5.43 \times \times 10^{-5}$ g, and of time 1.34×10^{-43} s. In addition to Planck's system, other natural systems were proposed, in which different universal constants are equated to unity. For example, in the system proposed by D. Hartree, the charge and mass of an electron, the radius of the first Bohr orbit, and the Planck constant divided by 2π (i.e., the constant $\hbar$) are all equated to unity. In this system the unit of time is 2.419×10^{-17} s, the unit of energy is 4.359×10^{-11} erg, etc.

The impossibility of equating to unity all the universal constants is due to the fact that there are definite relationships between some of them. For instance, the charge of an electron, the Planck constant and the velocity of light form a dimensionless combination, the so-called fine-

structure constant

$$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137} \tag{9.47}$$

It follows from this value of α that in the Planck system the charge of an electron will be $\frac{1}{\sqrt{2\pi \cdot 137}}$. In the same way the values of the constants in the laws of Stefan-Boltzmann, Wien and others (see Appendix 4) will be given.

Although in natural systems all the units are quite far from the dimensions of quantities generally encountered in practical work, these systems are employed with great success in theoretical physics, since they result in an exceedingly great simplification of the fundamental equations, freeing them of superfluous factors.

It should be noted in conclusion that natural systems are sometimes considered as systems in which constants equated to unity are used as the basic units, and this allows us to construct a system of dimensions whose formulas will contain the conditional symbols of the dimensions of these constants.

APPENDIX 1

Logarithmic Units

In Chapter Six we considered logarithmic units describing sound intensity—the bel, its tenth fraction—the decibel, and the neper. The frequency characteristic of musical intervals was also constructed to a logarithmic scale. The use of a logarithmic scale, however, is not at all limited to acoustics. A physical quantity sometimes changes within such broad limits that it is practically impossible to show it using a linear scale. For example, in modern vacuum engineering during the process of evacuation of an apparatus the pressure of a gas may change from atmospheric to 10^{-9}-10^{-11} atm, and in some laboratory investigations to 10^{-13}-10^{-15} atm. It is useless to attempt to show how this process proceeds with time in a linear scale of pressures. Similar instances are encountered in astronomy and in many other fields of physics and related sciences. The use of a logarithmic scale permits processes and laws to be shown with a practically unlimited range of the values of the quantity we are interested in, and both small and great values will be shown with sufficient clarity.

The reasons for using a logarithmic scale, however, are much more numerous. Quite often the essence of a phenomenon itself points to the expediency of describing it with the aid of logarithmic units. We have already mentioned the logarithmic nature of the psychophysical perception of the loudness and pitch of sound. This also relates, to the same extent, to the perception of other external stimuli that comply satisfactorily with the Weber-Fechner law, according to which perception is proportional to the logarithm of the stimulus.

For each stimulus there exists a minimum ratio of two values of the quantity characterizing it that can be detected by the corresponding sensory organ. For example, two brightnesses can be distinguished by the human eye if their ratio is about 2.5. This ratio determined the logarithmic scale for measuring the "brightness" of stars, the "stellar magnitude". We have put the word "brightness" in quotation marks because, owing to the great remoteness of stars, what is actually meant here is the illumination created by a given star at the boundary of the atmosphere. For this reason the human eye perceives stars as luminescent points of different brightness. Photoelectric registration makes it possible to introduce fractional values of stellar magnitudes. The brightest stars and, naturally, the Moon and the Sun, are described by negative values of the stellar magnitude (—12.54 and —26.59 stellar magnitude).

Another field of application of a logarithmic scale are processes in which a change in a quantity is proportional to the quantity itself. Such processes include the absorption of light by a homogeneous medium, the aperiodic discharge of a capacitor onto a resistor, the

attenuation of a signal along a transmission line, and a chemical or nuclear chain reaction. In the first examples the relevant quantity decreases with distance or time, and in the last one it grows. In the general form the law of change of the corresponding quantity can be written as

$$A = A_0 a^{kz} \tag{A.1}$$

where A_0 = initial value of a given quantity
A = its value at a value of the argument (distance, time, etc.) equal to z
k = factor describing the "rate" of the given process (absorption, attenuation, amplification, etc.)
a = base of logarithms used to describe the given process.

The factor k may be either negative (absorption, attenuation) or positive (amplification, development). The value of a is quite frequently taken equal to the base of natural logarithms e, but any other number may be used for this purpose, for example 10 or 2, the factor k being selected accordingly.

To characterize the pitch of sound, as indicated in Chapter Six, a logarithmic scale with a base 2 is employed. The same scale is used to describe radioactive decay when the half-life is used as a unit of time. The level of sound intensity is measured either in bels (with 10 as the base of logarithms), or in decibels (the base of logarithms is $\sqrt[10]{10} = 1.259+$), or in nepers (the base of logarithms is the number $e = 2.718+$).

The spreading of the use of logarithmic units was not smooth, but was accompanied by certain confusion. Whereas bels, decibels and nepers served to measure the difference in sound intensity levels in acoustics, in electrical and radio engineering when describing attenuation along a transmission line decibels were used to measure the power level, and nepers the change in the field intensity level. Since the power is proportional to the square of the field intensity amplitude, then for the ratio of two powers we have

$$\frac{P_1}{P_2} = \frac{E_1^2}{E_2^2} \tag{A.2}$$

or, using logarithms,

$$\log_{10} \frac{P_1}{P_2} = 2 \log_{10} \frac{E_1}{E_2} = 2 \times 0.4343 \log_e \frac{E_1}{E_2} \tag{A.3}$$

For this reason, if an acoustics 1 B was equal to 2.303 n [see formula (6.16)] or 1 dB = 0.2303 n, then in electrical engineering 1 dB = = 0.1151 n or 1 n = 8.686 dB.

The dual nature of logarithmic units partly spreads to the decibels themselves, which were begun to be used to measure both a change in power, and a change in field intensity, voltage, etc. This confusion led to the idea of introducing a common logarithmic unit whose use should be accompanied each time by an indication of the quantity it relates to. A number of proposals were made on the nature and name of this unit. The most favoured was a unit named the *decilog*, nume-

rically coinciding with the decibel, but applied to any quantity with the corresponding indication made. The use of decilogs made it possible to substitute addition and subtraction for multiplication and division, and even the final result could be expressed directly in decilogs. With respect to the decibel, it was decided to retain it only for measuring power levels. With such a definition of the decilog it can be said, for example, that one decilog of current intensity is equal to one decilog of voltage minus one decilog of resistance. According to what has been said above, the decilog can be defined either as 10 common logarithms of the given quantity, or as the logarithm of this quantity with the base $\sqrt[10]{10}$. When writing down quantities measured in decilogs, a subscript should be used indicating the unit in question. For instance, power measured in kilowatts and written down in decilogs should be designated $\mathrm{dlg}_{\mathrm{kW}}$.

Let us illustrate the above with an example, and determine the power of a current at a voltage of 2 kV and a current of 10 A:

$$x = \mathrm{dlg}_{\mathrm{kW}} = 3.01\ \mathrm{dlg}_{\mathrm{kV}} + 10\ \mathrm{dlg}_{\mathrm{A}} = 13.01\ \mathrm{dlg}_{\mathrm{kW}}$$

A special binary logarithmic unit, the *bit* (the name is derived from the words *b*inary dig*it*), employed in the theory of information, stands somewhat apart. If the given information is determined from a possible number n of equally probable events, then the measure of this information is given by the expression

$$N = \log_2 n \tag{A.4}$$

Let us consider the following problem as an example. Suppose we have a deck of 32 cards containing only "number" cards from 1 (ace) to eight. How many questions will have to be asked with the answer being only "yes" or "no" to guess a card that someone is thinking of? Each answer, obviously, will reduce the indeterminacy to one-half of the original number. Assume that the seven of spades is the card in question. The following scheme of questions and answers could be used, for example, to guess it:

Question	*Answer*
1. A black suit?	Yes
2. A club?	No
3. An even number?	No
4. An ace or three?	No
5. The seven of spades?	Yes

The only alternative to the last question is "The five of spades", and the answer "No" would show that the card in question is the seven of spades.

It can readily be seen that in any case five questions are sufficient to get the correct answer. This can be expressed by the statement that the knowledge of a definite card from among a total number of 32 contains information amounting to 5 bits. The knowing of the square of a chess-board on which a given figure is contains, obviously, $N = \log_2 64 = 6$ bits, etc.

APPENDIX 2

Measuring the Density of a Liquid with an Areometer

The density of a liquid can be measured with the aid of an areometer. A schematic view of such an instrument is shown in Fig. 30. Its lower part is provided with weight *1* that keeps it in a vertical position when immersed in the liquid being measured. Narrow tube *2* is calibrated according to the density being measured, since the depth of submergence depends on the density of the liquid. At present these calibrations directly show the density (usually in grams per cubic centimetre). Previously, however, conditional scales were used and the density was determined in degrees of the relevant scale. The calibrations were marked on the tube at equal distances from one another. If an areometer is immersed up to the mark conditionally taken as zero into a liquid with a density ρ_0, the volume of its submerged part being V, and then immersed into a liquid with a density ρ, the level changing by n divisions and the volume of the narrow tube between two divisions being v, then

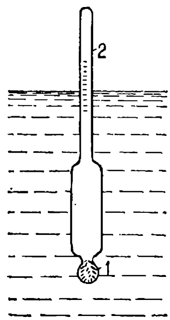

Fig. 30

$$\rho = \rho_0 \frac{V}{V \pm nv} \tag{A.5}$$

where the plus sign corresponds to a lighter and the minus sign to a heavier liquid. A definite relationship $N = V/v$ was established for different areometers. Here the following formula will serve to convert density into degrees (divisions of the scale) or into relative density, with the density ρ_0 taken as unity

$$\rho = \frac{N}{N \pm n} \tag{A.6}$$

In the greatest favour was the Baumé areometer (hydrometer), in which $N = 144.3$, and the density of water at 15°C was taken as the unit of density. The density was obtained in Baumé degrees (°Be).

APPENDIX 3

pH Index

The activity of electrolyte solutions depends on the concentration of ions in them. This relationship, however, is not quite single-valued owing to the interaction between ions. For this reason concentration can serve to describe the activity of a solution only when it is greatly diluted. At high concentrations the concept of equivalent concentration is introduced, which is the product of the actual concentration and an activity factor less than unity. Since both the actual and the equivalent concentrations of ions can change within very broad limits, a logarithmic scale is used. The index measured in this scale (designated pH) is equal to the common logarithm, with sign reversed, of the activity or equivalent concentration of hydrogen ions measured in gram-equivalents per litre. Since the concentration of hydrogen ions in water (and chemically neutral solutions) is 10^{-7}, then for water $pH = 7$. In acid solutions the hydrogen ion concentration is higher, and accordingly $pH < 7$, and in alkaline solutions, on the contrary, $pH > 7$.

APPENDIX 4

Constants

The present appendix gives the values of the most important universal constants. Those of them whose meaning is sufficiently obvious are given without explanations. For others either a reference to the corresponding formulas in the present book is given, or the origin and physical meaning of the constant are explained. In addition, since some of the constants are interrelated, the formulas are given showing the relationships between them.

The numerical values of the constants are given with such a number of digits that in the event of their more accurate determination, the change will be not more than by unity in the next to last significant digit. All values are given in the SI and cgs systems, and in individual cases in some non-system units.

Velocity of light

$$c = 2.997925 \times 10^{8} \text{ m/s} = 2.997925 \times 10^{10} \text{ cm/s}$$

Since a number of expressions, in particular some equations of electromagnetism in the cgs system, contain the velocity of light to the powers 2, —1 and —2, we give the corresponding values with an accuracy of unity in the last digit:

$$c^2 = 8.98726 \times 10^{16} \text{ m}^2/\text{s}^2 = 8.98726 \times 10^{20} \text{ cm}^2/\text{s}^2$$

$$\frac{1}{c} = 3.33572 \times 10^{-9} \text{ s/m} = 3.33572 \times 10^{-11} \text{ s/cm}$$

$$\frac{1}{c^2} = 1.11270 \times 10^{-17} \text{ s}^2/\text{m}^2 = 1.11270 \times 10^{-21} \text{ s}^2/\text{cm}^2$$

Avogadro's number— the number of molecules in a kilomole or mole

$$N_A = 6.0225 \times 10^{26} \text{ kmole}^{-1} = 6.0225 \times 10^{23} \text{ mole}^{-1}$$

Gravitational constant

$$G = 6.670 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2 = 6.670 \times 10^{-8} \text{ dyn}\cdot\text{cm}^2/\text{g}^2$$

Charge of an electron

$$e = 1.60210 \times 10^{-19} \text{ C} = 4.8030 \times 10^{-10} \text{ cgs}_Q$$

Mass of an electron

$$m_e = 9.1091 \times 10^{-31} \text{ kg} = 9.1091 \times 10^{-28} \text{ g}$$

Mass of a proton

$$m_p = 1.67252 \times 10^{-27} \text{ kg} = 1.67252 \times 10^{-24} \text{ g}$$

Mass of a neutron

$$m_n = 1.67482 \times 10^{-27}\ \text{kg} = 1.67482 \times 10^{-24}\ \text{g}$$

Faraday's constant (*faraday*)—the quantity of electricity which, when flowing through an electrolyte, causes one gram-equivalent of substance to be liberated at each electrode

$$F = eN_A = 9.6487 \times 10^4\ \text{C} = 2.8926 \times 10^{14}\ \text{cgs}_Q$$

Planck's constant

$$h = 6.62517 \times 10^{-34}\ \text{J·s} = 6.62517 \times 10^{-27}\ \text{erg·s}$$

$$\hbar = \frac{h}{2\pi} = 1.0544 \times 10^{-34}\ \text{J·s} = 1.0544 \times 10^{-27}\ \text{erg·s}$$

The constant $\hbar$ is sometimes called the *Dirac constant.*

Fine-structure constant. The investigation of the spectrum lines of hydrogen with the aid of instruments having a high resolution has shown that these lines possess a fine structure, i.e., consist of several lines very close to one another.

The fine structure of the lines is explained when account is taken of the theory of relativity and the own magnetic moment of an electron. The additional energy causing the splitting of the lines is determined by an expression including the dimensionless factor α called the fine-structure constant and numerically equal to

$$\alpha = \frac{e^2}{\hbar c} = 7.2970 \times 10^{-3}$$

Its reciprocal is

$$\frac{1}{\alpha} = 137.039$$

Ratio of the charge of an electron to its mass

$$\frac{e}{m_e} = 1.75880 \times 10^{11}\ \text{C/kg} = 5.27274 \times 10^{17}\ \text{cgs}_Q/\text{g}$$

Compton wavelength. When X-rays are scattered on free electrons the wavelength changes, owing to the exchange of energy and impulse between a photon and an electron (the Compton effect). This change is determined by the formula

$$\Delta\lambda = \lambda_0 (1 - \cos\theta) \tag{A.7}$$

where θ is the angle of deviation of a photon from its initial direction, and λ_0 is the Compton wavelength:

$$\lambda_0 = \frac{h}{m_e c} = 2.42621 \times 10^{-12}\ \text{m} = 2.42621 \times 10^{-10}\ \text{cm}$$

Sometimes a quantity obtained by dividing λ_0 by 2π is used in equations:

$$\lambda = \frac{\lambda_0}{2\pi} = 3.86144 \times 10^{-13}\ \text{m} = 3.86144 \times 10^{-11}\ \text{cm}$$

Rydberg constant. Formula (9.34) determines the wave numbers of spectrum lines of atoms similar to hydrogen. The Rydberg con-

stant R in this formula changes somewhat for different atoms owing to the difference in the masses of their nuclei. For an infinitely massive nucleus

$$R_\infty = \frac{m_e e^4}{4\pi\hbar^3 c} = 1.0973731 \times 10^7 \text{ m}^{-1} = 1.0973731 \times 10^5 \text{ cm}^{-1}$$

Bohr radius is the radius of the orbit of an electron in a hydrogen atom in the normal state according to Bohr's "classical" theory

$$a_0 = \frac{\hbar^2}{m_e e^2} = 5.29187 \times 10^{-11} \text{ m} = 5.29187 \times 10^{-9} \text{ cm} = 0.529187 \text{ Å}$$

Bohr magneton. A definition of the Bohr magneton was given in Sec. 9.2 [formula (9.6)]. Its more accurate value in the customarily used units is

$$\mu_B = 9.2732 \times 10^{-24} \text{ J}\cdot\text{T}^{-1} = 9.2732 \times 10^{-21} \text{ erg}\cdot\text{Gs}^{-1}$$

Standard volume of a gas is the volume of a kilomole or mole of the gas in standard conditions (0°C and 1 atm):

$$V_0 = 22.414 \text{ m}^3/\text{kmole (l/mole)} = 2.2414 \times 10^3 \text{ cm}^3/\text{mole}$$

Universal gas constant. According to the Clapeyron-Mendeleev equation (5.2), the universal gas constant can be determined by the expression

$$R = \frac{pV_0}{T} \tag{A.8}$$

Upon inserting the values of p and T corresponding to standard conditions, we get the value of R:

$$R = 8.3143 \times 10^3 \text{ J/(kmole}\cdot\text{deg)} = 9.3143 \times 10^7 \text{ erg/(mole}\cdot\text{deg)} =$$
$$= 1.9858 \text{ cal/(mole}\cdot\text{deg)} = 8.2053 \times 10^{-2} \text{ l}\cdot\text{atm/(mole}\cdot\text{deg)}$$

Boltzmann constant can be determined as the ratio of the universal gas constant to Avogadro's number

$$k = \frac{R}{N_A} = 1.3805 \times 10^{-23} \text{ J/deg} = 1.3805 \times 10^{-16} \text{ erg/deg}$$

Constant in the Stefan-Boltzmann law [formulas (5.4) and (8.11)]

$$\sigma = 5.669 \times 10^{-8} \text{ W/(m}^2\cdot\text{deg}^4) = 5.669 \times 10^{-5} \text{ erg/(cm}^2\cdot\text{deg}^4)$$

Constant in the Wien displacement law [formula (5.5)]

$$b = 2.8978 \times 10^{-3} \text{ m}\cdot\text{deg} = 0.28978 \text{ cm}\cdot\text{deg}$$

The *Planck radiation law* makes it possible to express the constants σ and b through the constants h, k and c:

$$\sigma = 1.0823 \frac{12\pi k^4}{c^2 h^3} \tag{A.9}$$

$$b = \frac{ch}{4.9651k} \tag{A.10}$$

(the factors 1.0823 and 4.9651 are obtained as a result of the corresponding mathematical operations).

APPENDIX 5

Tables

Table 1 is a summary of the units of geometrical and mechanical quantities. Table 24 is a similar summary of electrical and magnetic units. The units and their dimensions are given in four systems. Table 25 gives the equations of electromagnetism, also in four systems. Tables 2-23 and 26-45 are conversion tables for units of various quantities in different systems, and partly for non-system units. The conversion factors are given, as a rule, with an accuracy to the third significant digit, an accuracy to the fourth digit being used only for the units of time. Tables 46-53 are auxiliary ones and contain some British and U.S. units, units not approved by the relevant standards, scales of hardness and wind velocity, symbols of units and decimal prefixes, etc. Tables 54-59 are purely illustrative and have been included to show the order of the values obtained when measuring some properties of materials in units of different systems or in the most widespread non-system units.

LIST OF TABLES

TABLE 1. **Summary Table of Geometrical and Mechanical Units**

Quantity	Formula	Dimension formula		Units		
		SI and cgs	mk(force)s	SI	cgs	mk(force)s
Length	l	L	L	m	cm	m
Mass	$m=\frac{f^*}{a}$	M	$L^{-1}FT^2$	kg	g	i (tum)
Time	t	T	T	s	s	s
Area	$A=l^2$	L^2	L^2	m^2	cm^2	m^2
Volume	$V=l^3$	L^3	L^3	m^3	cm^3	m^3
Angle	$\varphi=\frac{l}{r}$	1	1	rad	rad	rad
Solid angle	$\Omega=\frac{A}{r^2}$	1	1	sr	sr	sr
Curvature	$\rho=\frac{1}{r}$	L^{-1}	L^{-1}	m^{-1}	cm^{-1}	m^{-1}
Gaussian curvature	$K=\frac{1}{r^2}$	L^{-2}	L^{-2}	m^{-2}	cm^{-2}	m^{-2}
Statical moment of plane figures	$S_z=\int_A r\,dA$	L^3	L^3	m^3	cm^3	m^3
Axial and polar moments of plane figures	$I_z=\int_A r^2dA$	L^4	L^4	m^4	cm^4	m^4

Table 1 (continued)

Quantity	Formula	Dimension formula		Units		
		SI and cgs	mk(force)s	SI	cgs	mk(force)s
Velocity	$v=\frac{l}{t}$	LT^{-1}	LT^{-1}	m/s	cm/s	m/s
Acceleration	$a=\frac{v_2-v_1}{t}$	LT^{-2}	LT^{-2}	m/s^2	cm/s^2	m/s^2
Angular velocity	$\omega=\frac{\varphi}{t}$	T^{-1}	T^{-1}	s^{-1}	s^{-1}	s^{-1}
Angular acceleration	$\alpha=\frac{\omega_2-\omega_1}{t}$	T^{-2}	T^{-2}	s^{-2}	s^{-2}	s^{-2}
Period	$T=\frac{2\pi}{\omega}$	T	T	s	s	s
Frequency	$\nu=\frac{1}{T}$	T^{-1}	T^{-1}	Hz	Hz	Hz
Velocity gradient	grad $v=\frac{dv}{dl}$	T^{-1}	T^{-1}	s^{-1}	s^{-1}	s^{-1}
Volumetric flow rate	$Q_v=\frac{dV}{dt}$	L^3T^{-1}	L^3T^{-1}	m^3/s	cm^3/s	m^3/s
Volumetric flow rate density	$q_V=\frac{Q_v}{A}$	LT^{-1}	LT^{-1}	m/s	cm/s	m/s
Force	$f=ma$ **	LMT^{-2}	F	N	dyn	kgf

Table 1 (concluded)

Quantity	Formula	Dimension formula		Units		
		SI and cgs	mk(force)s	SI	cgs	mk(force)s
Modulus of elasticity and shear	$E=\frac{fl}{A\Delta l}$	$L^{-1}MT^{-2}$	$L^{-2}F$	N/m²	dyn/cm²	kgf/m²
Coefficient of bulk compression	$k=-\frac{1}{V}\frac{dV}{dp}$	$LM^{-1}T^{2}$	$L^{2}F^{-1}$	m²/N	cm²/dyn	m²/kgf
Viscosity	$\eta=-\frac{f}{A\frac{dv}{dl}}$	$L^{-1}MT^{-1}$	$L^{-2}FT$	N·s/m²	P	kgf·s/m²
Coefficient of surface tension	$\sigma=\frac{f}{l}$	MT^{-2}	$L^{-1}F$	N/m (J/m²)	dyn/cm (erg/cm²)	kgf/m
Diffusion coefficient	$D=-\frac{\Delta m}{\Delta tA\frac{d\rho}{dl}}$	$L^{2}T^{-1}$	$L^{2}T^{-1}$	m²/s	cm²/s	m²/s

* In mk(force)s system.
** In SI and cgs systems.

Table 1 (continued)

Quantity	Formula	Dimension formula		Units		
		SI and cgs	mk(force)s	SI	cgs	mk(force)s
Moment of force	$M=fl$	L^2MT^{-2}	LF	n·m	dyn·cm	kgf·m
Impulse	ft	LMT^{-1}	FT	N·s	dyn·s	kgf·s
Momentum	mv	LMT^{-1}	FT	kg·m/s	g·cm/s	i·m/s
Work and energy	$W=fl\cos(\widehat{f,l})$	L^2MT^{-2}	LF	J	erg	kgf·m
Energy density	$e=\frac{E}{V}$	$L^{-1}MT^{-2}$	$L^{-2}F$	J/m^3	erg/cm^3	kgf/m^2
Power	$P=\frac{W}{t}$	L^2MT^{-3}	LFT^{-1}	W	erg/s	kg·m/s
Impulse of moment of force	Mt	L^2MT^{-1}	LFT	N·s·m	dyn·s·cm	kgf·s·m
Moment of momentum (angular momentum)	$\mathscr{L}=mvr=I\omega$	L^2MT^{-1}	LFT	kg·m^2/s	g·cm^2/s	i·m^2/s
Pressure	$p=\frac{f}{A}$	$L^{-1}MT^{-2}$	$L^{-2}F$	N/m^2	dyn/cm^2	kgf/m^2
Pressure gradient	$\operatorname{grad} p=\frac{dp}{dl}$	$L^{-2}MT^{-2}$	$L^{-3}F$	N/m^3	dyn/cm^3	kgf/m^3
Moment of inertia of bodies	$I=\int_V r^2dm$	L^2M	LFT^2	kg·m^2	g·cm^2	i·m^2 (kgf·m·s^2)
Density	$\rho=\frac{m}{V}$	$L^{-3}M$	$L^{-4}FT^2$	kg/m^3	g/cm^3	i/m^3 (kgf·s^2/m^4)

TABLE 2. **Conversion Table—Length**

	km	m	cm	mm	μ
1 km =	1	10^3	10^5	10^6	10^9
1 m =	10^{-3}	1	10^2	10^3	10^6
1 cm =	10^{-5}	10^{-2}	1	10	10^4
1 mm =	10^{-6}	10^{-3}	10^{-1}	1	10^3
1 μ =	10^{-9}	10^{-6}	10^{-4}	10^{-3}	1
1 nm =	10^{-12}	10^{-9}	10^{-7}	10^{-6}	10^{-3}
1 Å =	10^{-13}	10^{-10}	10^{-8}	10^{-7}	10^{-4}
1 XU =	10^{-16}	10^{-13}	10^{-11}	10^{-10}	10^{-7}
1 inch =	2.54×10^{-5}	2.54×10^{-2}	2.54	25.4	2.54×10^4
1 foot =	3.05×10^{-4}	0.305	30.5	3.05×10^2	3.05×10^5
1 mile (nautical) =	1.85	1.85×10^3	1.85×10^5	1.85×10^6	1.85×10^9

TABLE 3. **Conversion Table—Area**

	km^2	ha	a	m^2
1 km^2 =	1	100	10^4	10^6
1 ha =	10^{-2}	1	10^2	10^4
1 a =	10^{-4}	10^{-2}	1	10^2
1 m^2 =	10^{-6}	10^{-4}	10^{-2}	1
1 cm^2 =	10^{-10}	10^{-8}	10^{-6}	10^{-4}
1 mm^2 =	10^{-12}	10^{-10}	10^{-8}	10^{-6}
1 sq. inch =	6.45×10^{-10}	6.45×10^{-8}	6.45×10^{-6}	6.45×10^{-4}
1 sq. foot =	9.29×10^{-8}	9.29×10^{-6}	9.29×10^{-4}	9.29×10^{-2}
1 acre =	4.05×10^{-3}	0.405	40.5	4.05×10^3
1 sq. mile (nautical) =	3.43	3.43×10^2	3.43×10^4	3.43×10^6

nm	Å	XU	inch	foot	mile (nautical)
10^{12}	10^{13}	10^{16}	3.94×10^{4}	3.28×10^{3}	0.540
10^{9}	10^{10}	10^{13}	39.4	3.28	5.40×10^{-4}
10^{7}	10^{8}	10^{11}	0.394	3.28×10^{-2}	5.40×10^{-6}
10^{6}	10^{7}	10^{10}	3.94×10^{-2}	3.28×10^{-3}	5.40×10^{-7}
10^{3}	10^{4}	10^{7}	3.94×10^{-5}	3.28×10^{-6}	5.40×10^{-10}
1	10	10^{4}	3.94×10^{-8}	3.28×10^{-9}	5.40×10^{-13}
0.1	1	10^{3}	3.94×10^{-9}	3.28×10^{-10}	5.40×10^{-14}
10^{-4}	10^{-3}	1	3.94×10^{-12}	3.28×10^{-13}	5.40×10^{-17}
2.54×10^{7}	2.54×10^{8}	2.54×10^{11}	1	8.33×10^{-2}	1.37×10^{-5}
3.05×10^{8}	3.05×10^{9}	3.05×10^{12}	12	1	1.65×10^{-4}
1.85×10^{12}	1.85×10^{13}	1.85×10^{16}	7.29×10^{4}	6.08×10^{3}	1

cm^2	mm^2	sq. inch	sq. foot	acre	sq. mile (nautical)
10^{10}	10^{12}	1.55×10^{9}	1.08×10^{7}	2.47×10^{2}	0.292
10^{8}	10^{10}	1.55×10^{7}	1.08×10^{5}	2.47	2.92×10^{-3}
10^{6}	10^{8}	1.55×10^{5}	1.08×10^{3}	2.47×10^{-2}	2.92×10^{-5}
10^{4}	10^{6}	1.55×10^{3}	10.8	2.47×10^{-4}	2.92×10^{-7}
1	10^{2}	0.155	1.08×10^{-3}	2.47×10^{-8}	2.92×10^{-11}
10^{-2}	1	1.55×10^{-3}	1.08×10^{-5}	2.47×10^{-10}	2.92×10^{-13}
6.45	6.45×10^{-2}	1	6.94×10^{-3}	1.59×10^{-7}	1.88×10^{-10}
9.29×10^{2}	9.29×10^{4}	1.44×10^{2}	1	2.30×10^{-5}	2.71×10^{-8}
4.05×10^{7}	4.05×10^{9}	6.27×10^{6}	4.36×10^{4}	1	1.18×10^{-3}
3.43×10^{10}	3.43×10^{12}	5.32×10^{9}	3.69×10^{7}	8.47×10^{2}	1

TABLE 4. **Conversion Table—Volume**

	m^3	l (dm^3)	cm^3	mm^3	cu. inch	cu. foot	1 UK gallon
1 m^3 =	1	10^3	10^6	10^9	6.10×10^4	35.3	2.20×10^2
1 l (dm^3) =	10^{-3}	1	10^3	10^6	61	3.53×10^{-2}	0.220
1 cm^3 =	10^{-6}	10^{-3}	1	10^3	6.10×10^{-2}	3.53×10^{-5}	2.20×10^{-4}
1 mm^3 =	10^{-9}	10^{-6}	10^{-3}	1	6.10×10^{-5}	3.53×10^{-8}	2.20×10^{-7}
1 cu. inch =	1.64×10^{-5}	1.64×10^{-2}	16.4	1.6×10^4	1	5.79×10^{-4}	3.61×10^{-3}
1 cu. foot =	2.83×10^{-2}	28.3	2.83×10^4	2.83×10^7	1.73×10^3	1	6.23
1 UK gallon =	4.55×10^{-3}	4.55	4.55×10^3	4.55×10^6	2.77×10^2	0.161	1

TABLE 5. **Conversion Table—Solid Angle**

	sr	solid angle of complete sphere	right solid angle	$\square^\circ$ (square degree)
1 sr =	1	7.96×10^{-2}	0.637	3.28×10^3
1 solid angle of complete sphere =	12.6	1	8	4.13×10^4
1 right solid angle =	1.57	0.125	1	5.16×10^3
1 $\square^\circ$ (square degree) =	3.05×10^{-4}	2.42×10^{-5}	1.94×10^{-4}	1

TABLE 6. **Conversion Table—Angle**

	rad	°	′	″	rev	∟	g	c	cc
1 rad =	1	57.3	3.44×10^{3}	2.06×10^{5}	0.159	0.637	63.7	6.37×10^{3}	6.37×10^{5}
1° =	1.75×10^{-2}	1	60	3.6×10^{3}	2.78×10^{-3}	1.11×10^{-2}	1.11	1.11×10^{2}	1.11×10^{4}
1′ =	2.91×10^{-4}	1.67×10^{-2}	1	60	4.63×10^{-5}	1.85×10^{-4}	1.85×10^{-2}	1.85	1.85×10^{2}
1″ =	4.85×10^{-6}	2.78×10^{-4}	1.67×10^{-2}	1	7.72×10^{-7}	3.09×10^{-6}	3.09×10^{-4}	3.09×10^{-2}	3.09
1 revolution (circle) =	6.28	3.60×10^{2}	2.16×10^{4}	1.30×10^{6}	1	4	4×10^{2}	4×10^{4}	4×10^{6}
1 ∟ (right angle) =	1.57	90	5.40×10^{3}	3.24×10^{5}	0.25	1	10^{2}	10^{4}	10^{6}
1^{g} (gon) =	1.57×10^{-2}	0.900	54.0	3.24×10^{3}	2.5×10^{-3}	10^{-2}	1	10^{2}	10^{4}
1^{c} (metric min.) =	1.57×10^{-4}	9×10^{-3}	0.54	32.4	2.5×10^{-5}	10^{-4}	10^{-2}	1	10^{2}
1^{cc} (metric sec.) =	1.57×10^{-6}	9×10^{-5}	5.4×10^{-3}	0.324	2.5×10^{-7}	10^{-6}	10^{-4}	10^{-2}	1

TABLE 7. **Conversion Table—Time**

	s	min	h	day	week	year
1 s =	1	1.667×10^{-2}	2.778×10^{-4}	1.157×10^{-5}	1.653×10^{-6}	3.169×10^{-8}
1 min =	60	1	1.667×10^{-2}	6.944×10^{-4}	9.921×10^{-5}	1.901×10^{-6}
1 h =	3.6×10^{3}	60	1	4.167×10^{-2}	5.952×10^{-3}	1.141×10^{-4}
1 day =	8.64×10^{4}	1.44×10^{-3}	24	1	0.1429	2.738×10^{-3}
1 week =	6.048×10^{5}	1.008×10^{4}	168	7	1	1.915×10^{-2}
1 year =	3.156×10^{7}	5.259×10^{5}	8.766×10^{3}	365.2	52.18	1

TABLE 8. **Conversion Table—Velocity**

	m/s	m/min	cm/s	km/h	knot
1 m/s =	1	60	10^2	3.6	1.94
1 m/min =	1.67×10^{-2}	1	1.67	6×10^{-2}	3.24×10^{-2}
1 cm/s =	10^{-2}	0.6	1	3.6×10^{-2}	1.94×10^{-2}
1 km/h =	0.278	16.7	27.8	1	0.540
1 knot =	0.514	30.9	51.4	1.85	1

TABLE 9. **Conversion Table—Acceleration**

	m/s²	cm/s²	*g*
1 m/s^2 =	1	10^2	0.102
1 cm/s^2 =	10^{-2}	1	1.02×10^{-3}
1 *g* =	9.81	9.81×10^2	1

TABLE 10. **Conversion Table—Angular Velocity**

	rad/s	rev/s	rev/min	°/sec
1 rad/s =	1	0.159	9.55	57.3
1 rev/s (rps) =	6.28	1	60	3.6×10^2
1 rev/min (rpm) =	0.105	1.67×10^{-2}	1	6
1 °/s =	1.75×10^{-2}	2.78×10^{-3}	0.167	1

TABLE 11. **Conversion Table—Mass**

	kg	g	i (tum)	ton
1 kg =	1	10^3	0.102	10^{-3}
1 g =	10^{-3}	1	1.02×10^{-4}	10^{-6}
1 i (tum) =	9.81	9.81×10^3	1	9.8×10^{-3}
1 ton =	10^3	10^6	102	1

TABLE 12. **Conversion Table—Force**

	N	dyn	kgf	sn
1 N =	1	10^5	0.102	10^{-3}
1 dyn =	10^{-5}	1	1.02×10^{-6}	10^{-8}
1 kgf =	9.81	9.81×10^5	1	9.81×10^{-3}
1 sn (sthene) =	10^3	10^8	102	1

TABLE 13. Conversion Table—Pressure

	N/m²	dyn/cm²	kgf/m²	kgf/cm²	pieze	bar (hectopieze)	atm	mm Hg
1 N/m² (pascal) =	1	10	0.102	1.02×10^{-5}	10^{-3}	10^{-5}	9.87×10^{-6}	7.50×10^{-3}
1 dyn/cm² (μbar) =	0.1	1	1.02×10^{-2}	1.02×10^{-6}	10^{-4}	10^{-6}	9.87×10^{-7}	7.50×10^{-4}
1 kgf/m² (mm H_2O) =	9.81	98.1	1	10^{-4}	9.81×10^{-3}	9.81×10^{-5}	9.68×10^{-5}	7.35×10^{-2}
1 kgf/cm² (at) =	9.81×10^{4}	9.81×10^{5}	10^{4}	1	98.1	0.981	0.968	7.35×10^{2}
1 pieze =	10^{3}	10^{4}	1.02×10^{2}	1.02×10^{-2}	1	10^{-2}	9.87×10^{-3}	7.50
1 bar (hectopieze) =	10^{5}	10^{6}	1.02×10^{4}	10.2	10^{2}	1	0.987	7.5×10^{2}
1 atm =	1.01×10^{5}	1.01×10^{6}	1.03×10^{4}	1.03	1.01×10^{2}	1.01	1	7.6×10^{2}
1 mm Hg =	1.33×10^{2}	1.33×10^{3}	13.6	1.36×10^{-3}	0.133	1.33×10^{-2}	1.31×10^{-3}	1

TABLE 14. Conversion Table—Work and Energy

	J	erg	kgf·m	cal	kcal	kWh	hp·h	l·atm
1 J =	1	10^{7}	0.102	0.239	$2,39\times10^{-4}$	2.78×10^{-7}	3.78×10^{-7}	9.87×10^{-3}
1 erg =	10^{-7}	1	1.02×10^{-8}	2.39×10^{-8}	2.39×10^{-11}	2.78×10^{-14}	3.78×10^{-14}	9.87×10^{-10}
1 kgf·m =	9.81	9.81×10^{7}	1	2.34	2.34×10^{-3}	2.72×10^{-6}	3.70×10^{-6}	9.68×10^{-2}
1 cal =	4.19	4.19×10^{7}	0.427	1	10^{-3}	1.16×10^{-6}	1.58×10^{-6}	4.13×10^{-2}
1 kcal =	4.19×10^{3}	4.19×10^{10}	4.27×10^{2}	10^{3}	1	1.16×10^{-3}	1.58×10^{-3}	41.3
1 kWh (kilowatt-hour) =	3.6×10^{6}	3.6×10^{13}	3.67×10^{5}	6.8×10^{5}	8.6×10^{2}	1	1.36	3.55×10^{4}
1 hp·h (horsepower·hour) =	2.65×10^{6}	2.65×10^{13}	2.75×10^{5}	6.32×10^{5}	6.32×10^{2}	0.736	1	2.61×10^{4}
1 l·atm =	1.01×10^{2}	1.01×10^{9}	10.3	24.2	2.42×10^{-2}	2.81×10^{-5}	3.83×10^{-5}	1

TABLE 15. **Conversion Table—Power**

		W	erg/s	kW	kgf·m/s	cal/s	kcal/h	hp
1 W	=	1	10^{7}	10^{-3}	0.102	0.239	0.860	1.36×10^{-3}
1 erg/s	=	10^{-7}	1	10^{-10}	1.02×10^{-8}	2.39×10^{-8}	8.60×10^{-8}	1.36×10^{-10}
1 kW	=	10^{3}	10^{10}	1	1.02×10^{2}	2.39×10^{2}	8.60×10^{-2}	1.36
1 kgf·m/s	=	9.81	9.81×10^{7}	9.81×10^{-3}	1	2.34	8.43	1.33×10^{-2}
1 cal/s	=	4.19	4.19×10^{7}	4.19×10^{-3}	0.427	1	3.60	5.69×10^{-3}
1 kcal/h	=	1.16	1.16×10^{7}	1.16×10^{-3}	0.119	0.278	1	1.58×10^{-3}
1 hp	=	7.36×10^{2}	7.36×10^{9}	0.736	75	175.5	6.32×10^{2}	1

TABLE 16. **Conversion Table—Moment of Inertia**

	$kg \cdot m^2$	$g \cdot cm^2$	$i \cdot m^2$
1 $kg \cdot m^2$ =	1	10^7	0.102
1 $g \cdot cm^2$ =	10^{-7}	1	1.02×10^{-8}
1 $i \cdot m^2$ ($tum \cdot m^2$) =	9.81	9.81×10^7	1

TABLE 17. **Conversion Table—Moduli of Elasticity and Shear**

	N/m^2	dyn/cm^2	kgf/m^2	kgf/cm^2	kgf/mm^2
1 N/m^2 =	1	10	0.102	1.02×10^{-5}	1.02×10^{-7}
1 dyn/cm^2 =	0.1	1	1.02×10^{-2}	1.02×10^{-6}	1.02×10^{-8}
1 kgf/m^2 =	9.81	98.1	1	10^{-4}	10^{-6}
1 kgf/cm^2 =	9.81×10^4	9.81×10^5	10^4	1	10^{-2}
1 kgf/mm^2 =	9.81×10^6	9.81×10^7	10^6	10^2	1

TABLE 18. **Conversion Table—Reduced Pressure and Concentration**

	m^{-3}	l^{-1}	cm^{-3}	mole/l ($kmole/m^3$)
1 N/m^2	2.66×10^{20}	2.66×10^{17}	2.66×10^{14}	4.42×10^{-7}
1 dyn/cm^2	2.66×10^{19}	2.66×10^{16}	2.66×10^{13}	4.42×10^{-8}
1 atm	2.69×10^{25}	2.69×10^{22}	2.69×10^{19}	4.46×10^{-2}
1 mm Hg =	3.54×10^{22}	3.54×10^{19}	3.54×10^{16}	5.87×10^{-5}

TABLE 19. **Conversion Table—Specific Heat**

	$J/kg \cdot deg$	$erg/g \cdot deg$	$kcal/kg \cdot deg$ ($cal/g \cdot deg$)
1 $J/kg \cdot deg$ =	1	10^4	2.39×10^{-4}
1 $erg/g \cdot deg$ =	10^{-4}	1	2.39×10^{-8}
1 $kcal/kg \cdot deg$ ($cal/g \cdot deg$) =	4.19×10^3	4.19×10^7	1

TABLE 20. **Conversion Table—Thermal Conductivity**

		W/m·deg	erg/cm·deg	kcal/m·h·deg	cal/cm·s·deg
1 W/m·deg	=	1	10^5	0.860	2.39×10^{-3}
1 erg/cm·s·deg	=	10^{-5}	1	8.60×10^{-6}	2.39×10^{-8}
1 kcal/m·h·deg	=	1.16	1.16×10^5	1	2.78×10^{-3}
1 cal/cm·s·deg	=	4.19×10^2	4.19×10^7	3.6×10^2	1

TABLE 21. **Conversion Table—Heat Transfer Coefficient**

		W/m²·deg	erg/cm²·s·deg	kcal/m²·h·deg	cal/cm²·s·deg
1 W/m²·deg	=	1	10^3	0.860	2.39×10^{-5}
1 erg/cm²·s·deg	=	10^{-3}	1	8.60×10^{-4}	2.39×10^{-8}
1 kcal/m²·h·deg	=	1.16	1.16×10^3	1	2.78×10^{-5}
1 cal/cm²·s·deg	=	4.19	4.19×10^7	3.60×10^4	1

TABLE 22. **Conversion Table—Frequency Interval**

		savart	octave	millioctave	cent
1 savart	=	1	3.32×10^{-3}	3.32	3.98
1 octave	=	301	1	1000	1200
1 millioctave	=	0.301	10^{-3}	1	1.2
1 cent	=	0.251	8.33×10^{-4}	0.833	1

TABLE 23. **Musical Intervals**

Name of tone	Name of interval relative to "C"	Natural scale			Tempered scale	
		Frequency relative to that of "C"	Interval in savarts	Interval in cents	Interval in savarts	Interval in cents
C	Unison	1	0	0	0	0
D	Major tone	9/8	51.1	204	50.2	200
E	Major third	5/4	96.9	386	100.4	400
F	Fourth	4/3	125.0	498	125.4	500
G	Fifth	3/2	176.1	702	175.6	700
A	Major sixth	5/3	221.9	884	225.8	900
H (*B*)	Seventh	15/8	273.0	1088	276.0	1100
C	Octave	2	301.0	1200	301.0	1200

TABLE 24. **Summary Table of Electrical and Magnetic Units**

Quantity	Symbol	Defining relationships in systems: SI and cgsm	Defining relationships in systems: cgs	Dimension: SI
Quantity of electricity (charge)	Q	$Q=It$	$Q=r\sqrt{f}\varepsilon_r$	TI
Intensity of electrical field	E	$E=\frac{f}{Q}$		$LMT^{-3}I^{-1}$
Electric displacement	D	$D=\varepsilon_0\varepsilon_r E$	$D=\varepsilon_r E$	$L^{-2}TI$
Electric flux	N_D	$N_D=DA$		TI
Potential	U	$U=\frac{E_p}{Q}$		$L^2MT^{-3}I^{-1}$
Dipole moment	$\vec{P}$	$\vec{P}=Ql$		LTI
Surface density of charge	σ	$\sigma=\frac{Q}{A}$		$L^{-2}TI$
Volume density of charge	ρ	$\rho=\frac{Q}{V}$		$L^{-3}TI$
Capacitance	C	$C=\frac{Q}{U}$		$L^{-2}M^{-1}T^4I^2$
Dielectric polarization	$\vec{p}$	$\vec{p}=\frac{p}{V}$		$L^{-2}TI$
Absolute permittivity	ε	$\varepsilon=\varepsilon_0\varepsilon_r$	ε_r	$L^{-3}M^{-1}T^4I^2$
Dielectric susceptibility	χ_e	$\chi_e=\varepsilon_0(\varepsilon_r-1)$	$\chi_e \quad \frac{\varepsilon_r-1}{4\pi}$	$L^{-3}M^{-1}T^4I^2$
Current intensity	I	$I=\sqrt{\frac{2\pi rf}{\mu_0\mu_r l}}$	$I=\frac{Q}{t}$	I

formulas in systems			Name and symbol of unit in systems	
cgsμ_0	cgs	cgs ε_0	SI	cgs
$L^{1/2}M^{1/2}\mu_0^{-1/2}$	$L^{3/2}M^{1/2}T^{-1}$	$L^{3/2}M^{1/2}T^{-1}\varepsilon_0^{1/2}$	coulomb (C)	—
$L^{1/2}M^{1/2}T^{-2}\mu_0^{1/2}$	$L^{-1/2}M^{1/2}T^{-1}$	$L^{-1/2}M^{1/2}T^{-1}\varepsilon_0^{-1/2}$	volt per metre	—
$L^{-3/2}M^{1/2}\mu_0^{-1/2}$	$L^{-1/2}M^{1/2}T^{-1}$	$L^{-1/2}M^{1/2}T^{-1}\varepsilon_0^{1/2}$	coulomb per sq. metre	—
$L^{1/2}M^{1/2}\mu_0^{-1/2}$	$L^{3/2}M^{1/2}T^{-1}$	$L^{3/2}M^{1/2}T^{-1}\varepsilon_0^{1/2}$	coulomb (C)	—
$L^{3/2}M^{1/2}T^{-2}\mu_0^{1/2}$	$L^{1/2}M^{1/2}T^{-1}$	$L^{1/2}M^{1/2}T^{-1}\varepsilon_0^{-1/2}$	volt (V)	—
$L^{3/2}M^{1/2}\mu_0^{-1/2}$	$L^{5/2}M^{1/2}T^{-1}$	$L^{5/2}M^{1/2}T^{-1}\varepsilon_0^{1/2}$	coulomb-metre	—
$L^{-3/2}M^{1/2}\mu_0^{-1/2}$	$L^{-1/2}M^{1/2}T^{-1}$	$L^{-1/2}M^{1/2}T^{-1}\varepsilon_0^{1/2}$	coulomb per sq. metre (C/m^2)	—
$L^{-5/2}M^{1/2}\mu_0^{-1/2}$	$L^{-3/2}M^{1/2}T^{-1}$	$L^{-3/2}M^{1/2}T^{-1}\varepsilon_0^{-1/2}$	coulomb per cu. metre (C/m^3)	—
$L^{-1}T^{2}\mu_0^{-1}$	L	$L\varepsilon_0$	farad (F)	centimetre (cm)
$L^{-3/2}M^{1/2}\mu_0^{-1/2}$	$L^{-1/2}M^{1/2}T^{-1}$	$L^{-1/2}M^{1/2}T^{-1}\varepsilon_0^{1/2}$	coulomb per sq. metre (C/m^2)	—
$L^{-2}T^{2}\mu_0^{-1}$	1	ε_0	farad per metre (F/m)	—
$L^{-2}T^{2}\mu_0^{-1}$	1	ε_0	farad per metre (F/m)	—
$L^{1/2}M^{1/2}T^{-1}\mu_0^{-1/2}$	$L^{3/2}M^{1/2}T^{-2}$	$L^{3/2}M^{1/2}T^{-2}\varepsilon_0^{1/2}$	ampere (A)	—

Quantity	Symbol	Defining relationships in systems		Dimension
		SI and cgsm	cgs	SI
Current density	J	$J = \frac{I}{A}$		$L^{-2}I$
Resistance	R	$R = \frac{U}{I}$		$L^2MT^{-3}I^{-2}$
Conductance	G	$G = \frac{1}{R}$		$L^{-2}M^{-1}T^3I^2$
Magnetic induction	B	$B = \frac{f}{Il}$	$B = c\frac{f}{Il}$	$MT^{-2}I^{-1}$
Magnetic flux	Φ	$\Phi = BA$		$L^2MT^{-2}I^{-1}$
Intensity of magnetic field	H	$H = \frac{B}{\mu_0\mu_r}$	$H = \frac{B}{\mu_r}$	$L^{-1}I$
Magnetic moment	$\vec{p}_m$	$\vec{p}_m = \frac{M}{B} = IA$	$\vec{p}_m = \frac{M}{B} = \frac{1}{c} IA$	L^2I
Magnetomotive force	F	$F = \Sigma I$	$F = \frac{1}{c} 4\pi \Sigma I$	I
Inductance and mutual inductance	$L\ (M)$	$L = \frac{\Psi}{I} = -\frac{\mathscr{E}_i}{dI/dt}$	$L = \frac{c\Psi}{I} = -\frac{c^2\mathscr{E}_i}{dI/dt}$	$L^2MT^{-2}I^{-2}$
Magnetization	J	$J = \frac{\vec{p}_m}{V}$		$L^{-1}I$
Absolute permeability	μ	$\mu = \mu_0\mu_r$	μ_r	$LMT^{-2}I^{-2}$
Magnetic susceptibility	χ_m	$\chi_m = \mu_r - 1$	$\chi_m = \frac{\mu_r - 1}{4\pi}$	1

Notes. 1. All defining relationships are given for the simplest cases — homogene-
2. The dimensions in the cgs μ_0 and cgs ε_0 systems differ from the dimen-
the dimensions μ_0 and ε_0.

Table 24 (continued)

formulas in systems			Name and symbol of unit in systems	
cgs μ_0	cgs	cgs ε_0	SI	cgs
$L^{-3/2}M^{1/2}T^{-1}\mu_0^{-1/2}$	$L^{-1/2}M^{1/2}T^{-2}$	$L^{-1/2}M^{1/2}T^{-2}\varepsilon_0^{1/2}$	ampere per sq. metre (A/m^2)	—
$LT^{-1}\mu_0$	$L^{-1}T$	$L^{-1}T\varepsilon_0^{-1}$	ohm (Ω)	—
$L^{-1}T\mu_0^{-1}$	LT^{-1}	$LT^{-1}\varepsilon_0$	siemens (S)	—
$L^{-1/2}M^{1/2}T^{-1}\mu_0^{1/2}$	$L^{-1/2}M^{1/2}T^{-1}$	$L^{-3/2}M^{1/2}\varepsilon_0^{-1/2}$	tesla (T)	gauss (Gs)
$L^{3/2}M^{1/2}T^{-1}\mu_0^{1/2}$	$L^{3/2}M^{1/2}T^{-1}$	$L^{1/2}M^{1/2}\varepsilon_0^{-1/2}$	weber (Wb)	maxwell (Mx)
$L^{-1/2}M^{1/2}T^{-1}\mu_0^{-1/2}$	$L^{-1/2}M^{1/2}T^{-1}$	$L^{1/2}M^{1/2}T^{-2}\varepsilon_0^{1/2}$	ampere per metre (A/m)	oersted (Oe)
$L^{5/2}M^{1/2}T^{-1}\mu_0^{-1/2}$	$L^{5/2}M^{1/2}T^{-1}$	$L^{7/2}M^{1/2}T^{-2}\varepsilon_0^{1/2}$	ampere-sq. metre (A·m^2)	—
$L^{1/2}M^{1/2}T^{-1}\mu_0^{-1/2}$	$L^{1/2}M^{1/2}T^{-1}$	$L^{3/2}M^{1/2}T^{-2}\varepsilon_0^{1/2}$	ampere (A) ampere-turn (At)	gilbert (Gb)
$L\mu_0$	L	$L^{-1}T^2\varepsilon_0^{-1}$	henry (H)	centimetre (cm)
$L^{-1/2}M^{1/2}T^{-1}\mu_0^{-1/2}$	$L^{-1/2}M^{1/2}T^{-1}$	$L^{1/2}M^{1/2}T^{-2}\varepsilon_0^{1/2}$	ampere per metre (A/m)	—
μ_0	1	$L^{-2}T^2\varepsilon_0^{-1}$	henry per metre (H/m)	—
1	1	1	—	—

ous fields, non-varying currents (except for the e.m.f. of induction), etc.
sion formulas of the same units in the cgse and cgsm systems by the addition of

TABLE 25. Equations of Electromagnetism in Different Systems of Units*

<table>
<tr><th></th><th>SI</th><th>cgsm</th><th>cgse</th><th>cgs</th></tr>
<tr><td>Coulomb's law</td><td>$f=\frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{\varepsilon_r r^2}$</td><td>$f=\frac{1}{\varepsilon_0}\frac{Q_1Q_2}{\varepsilon_r r^2}$</td><td colspan="2">$f=\frac{Q_1Q_2}{\varepsilon_r r^2}$</td></tr>
<tr><td>Force acting on a charge in an electric field</td><td colspan="4">$f=QE$</td></tr>
<tr><td>Relationship between field intensity and displacement</td><td colspan="2">$D=\varepsilon_0\varepsilon_r E$</td><td colspan="2">$D=\varepsilon_r E$</td></tr>
<tr><td>Gauss theorem (electric flux through a closed surface)</td><td>$N_D=\oint D\cos\times$ $\times(\widehat{D,n})\,dA=Q$</td><td colspan="3">$N=\oint D\cos(\widehat{D,n})\,dA=4\pi Q$</td></tr>
<tr><td>Field intensity of point charge</td><td>$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{\varepsilon_r r^2}$</td><td>$E=\frac{1}{\varepsilon_0}\frac{Q}{\varepsilon_r r^2}$</td><td colspan="2">$E=\frac{Q}{\varepsilon_r r^2}$</td></tr>
<tr><td>Field intensity in plane capacitor</td><td>$E=\frac{\sigma}{\varepsilon_0\varepsilon_r}$</td><td>$E=\frac{4\pi\sigma}{\varepsilon_0\varepsilon_r}$</td><td colspan="2">$E=\frac{4\pi\sigma}{\varepsilon_r}$</td></tr>
<tr><td>Field intensity in cylindrical capacitor (τ = charge density per unit of capacitor length)</td><td>$E=\frac{1}{2\pi\varepsilon_0}\frac{\tau}{\varepsilon_r r}$</td><td>$E=\frac{1}{\varepsilon_0}\frac{2\tau}{\varepsilon_r r}$</td><td colspan="2">$E=\frac{2\tau}{\varepsilon_r r}$</td></tr>
<tr><td>Field intensity on axis of dipole (it is assumed that $r\gg l$; l = arm of dipole)</td><td>$E=\frac{1}{2\pi\varepsilon_0}\frac{\vec{p}}{\varepsilon_r r^3}$</td><td>$E=\frac{1}{\varepsilon_0}\frac{2\vec{p}}{\varepsilon_r r^3}$</td><td colspan="2">$E=\frac{2\vec{p}}{\varepsilon_r r^3}$</td></tr>
<tr><td>Dipole moment</td><td colspan="4">$\vec{p}=Ql$</td></tr>
<tr><td>Force of interaction of two dipoles arranged along one axis ($r\gg l$; l = arm of dipole)</td><td>$f=-\frac{3}{2\pi\varepsilon_0}\frac{\vec{p}_1\vec{p}_2}{\varepsilon_r r^4}$</td><td>$f=-\frac{6}{\varepsilon_0}\frac{\vec{p}_1\vec{p}_2}{\varepsilon_r r^4}$</td><td colspan="2">$f=-\frac{6\vec{p}_1\vec{p}_2}{\varepsilon_r r^4}=\chi_e E$</td></tr>
<tr><td>Dielectric polarization</td><td colspan="4">$\vec{P}=\frac{\vec{p}}{V}=\chi_e E$</td></tr>
</table>

Relationship between permittivity and dielectric susceptibility	$\varepsilon = \varepsilon_0\varepsilon_r = \varepsilon_0 + \chi_e$	$\varepsilon = \varepsilon_0\varepsilon_r = \varepsilon_r + 4\pi\chi_e$	$\varepsilon_r = 1 + 4\pi\chi_e$
Relationship between field intensity and potential	$E = -\mathrm{grad}\, U$		
Potential of field of point charge	$U = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\varepsilon_r r}$	$U = \frac{1}{\varepsilon_0}\frac{Q}{\varepsilon_r r}$	$U = \frac{Q}{\varepsilon_r r}$
Potential inside cylindrical capacitor (U_1=potential, R_1= radius of internal cylinder)	$U = U_1 - Er \log_e \frac{r}{R_1}$		
Relationship between capacitance, charge and potential	$Q = CU$		
Capacitance of isolated sphere	$C = 4\pi\varepsilon_0\varepsilon_r R$	$C = \varepsilon_0\varepsilon_r R$	$C = \varepsilon_r R$
Capacitance of plane capacitor	$C = \frac{\varepsilon_0\varepsilon_r A}{l}$	$C = \frac{\varepsilon_0\varepsilon_r A}{4\pi l}$	$C = \frac{\varepsilon_r A}{4\pi l}$
Capacitance of cylindrical capacitor	$C = \frac{2\pi\varepsilon_0\varepsilon_r l}{\log_e R_2/R_1}$	$C = \frac{\varepsilon_0\varepsilon_r l}{2 \log_e R_2/R_1}$	$C = \frac{\varepsilon_r l}{2 \log_e R_2/R_1}$
Capacitance of two-conductor line (l=length of line, R=radius of conductors, a = distance between conductors; $a \gg R$)	$C = \frac{\pi\varepsilon_0\varepsilon_r l}{\log_e a/R}$	$C = \frac{\varepsilon_0\varepsilon_r l}{4\pi \log_e a/R}$	$C = \frac{\varepsilon_r l}{4\pi \log_e a/R}$
Energy of charged conductor	$\mathbf{E} = \frac{QU}{2} = \frac{U^2C}{2} = \frac{Q^2}{2C}$		
Volume density of energy of electric field	$u = \frac{ED}{2} = \frac{\varepsilon_0\varepsilon_r E^2}{2} = \frac{D^2}{2\varepsilon_0\varepsilon_r}$	$u = \frac{ED}{8\pi} = \frac{\varepsilon_0\varepsilon_r E^2}{8\pi} = \frac{D^2}{8\pi\varepsilon_0\varepsilon_r}$	$u = \frac{ED}{8\pi} = \frac{\varepsilon_r E^2}{8\pi} = \frac{D^2}{8\pi\varepsilon_r}$
Definition of conductivity current	$I = \frac{dQ}{dt}$		

Table 25 (continued)

	SI	cgsm	cgse	cgs
Ohm's law	$I = \frac{U}{R}$			
Power of current	$P = UI$			
Force acting on element of current in a magnetic field (Ampere's formula)	$df = \mu_0\mu_r HI\,dl \times \sin(\widehat{H, dl})$	$df = \mu_r HI\,dl \times \sin(\widehat{H, dl})$	$df = \mu_0\mu_r HI\,dl \times \sin(\widehat{H, dl})$	$df = \frac{\mu_r}{c} HI\,dl \times \sin(\widehat{H, dl})$
Moment acting on circuit with current in a magnetic field	$M = B\vec{p}m \sin(\widehat{B, \vec{p}m})$			
Magnetic moment of circuit with current	$\vec{p}m = IA$			$\vec{p}m = \frac{1}{c} IA$
Work of displacement of circuit with current in a magnetic field	$W = I\Delta\Psi$			$W = \frac{1}{c} I\Delta\Psi$
Biot, Savart and Laplace law	$H = \frac{1}{4\pi} \times \oint \frac{I\,dl \sin(\widehat{r, dl})}{r^2}$	$H = \oint \frac{I\,dl \sin(\widehat{r, dl})}{r^2}$		$H = \frac{1}{c} \times \oint \frac{I\,dl \sin(\widehat{r, dl})}{r^2}$
Relationship between induction and intensity of magnetic field	$B = \mu_0\mu_r H$	$B = \mu_r H$	$B = \mu_0\mu_r H$	$B = \mu_r H$
Intensity of magnetic field of infinitely long straight conductor with current	$H = \frac{I}{2\pi r}$	$H = \frac{2I}{r}$		$H = \frac{1}{c} \frac{2I}{r}$
Intensity of field at centre of ring with current	$H = \frac{I}{2R}$	$H = \frac{2\pi I}{R}$		$H = \frac{1}{c} \frac{2\pi I}{R}$
Intensity of field on axis of long solenoid (N=total num-				

ber of turns, N_0 = number of turns per unit of length)	$H = I\frac{N}{l} = IN_0$	$H = 4\pi I\frac{N}{l} = 4\pi IN_0$		$H = \frac{1}{c}4\pi IN_0$
Force of interaction of two parallel currents	$f = \frac{\mu_0\mu_r}{2\pi}\frac{I_1I_2l}{2}$	$f = \mu_r\frac{2I_1I_2l}{r}$	$f = \mu_0\mu_r\frac{2I_1I_2l}{r}$	$f = \frac{\mu_r}{c^2}\frac{2I_1I_2l}{r}$
Relationship between magnetic flux and magnetic induction		$d\Phi = B\,dA\cos(\widehat{B, n})$		
Magnetomotive force	$F = \Sigma I$	$F = 4\pi\Sigma I$		$F = \frac{1}{c}4\pi\Sigma I$
Magnetization		$J = \frac{\vec{p}_m}{V} = \chi_m H$		
Relationship between permeability and magnetic susceptibility	$\mu_r = 1 + \chi_m$	$\mu_r = 1 + 4\pi\chi_m$		
Relationship between flux linkage, current and inductance of circuit		$\Psi = LI$		$\Psi = \frac{1}{c}LI$
Inductance of solenoid (A = cross-sectional area of solenoid, N = total number of turns, N_0 = number of turns per unit of length)	$L = \mu_0\mu_r\frac{N^2S}{l} = \mu_0\mu_rN_0^2V$	$L = 4\pi\mu_r\frac{N^2A}{l} = 4\pi\mu_rN_0V$	$L = 4\pi\mu_0\mu_r\frac{N^2A}{l} = 4\pi\mu_0\mu_rN_0^2V$	$L = 4\pi\mu_r\frac{N^2A}{l} = 4\pi\mu N_0^2V$
Inductance of two-conductor line	$L = \mu_0\frac{l}{\pi}\times\log_e\frac{a}{R}$	$L = 4l\log_e\frac{a}{R}$	$L = 4\mu_0 l\log_e\frac{a}{R}$	$L = 4l\log_e\frac{a}{R}$
Electromotive force of induction		$\varepsilon_i = -\frac{d\Psi}{dt}$		$\varepsilon_i = -\frac{1}{c}\frac{d\Psi}{dt}$
Electromotive force of self-induction		$\varepsilon_{si} = -L\frac{dI}{dt}$		$\varepsilon_{si} = -\frac{1}{c^2}\frac{dI}{dt}$

Table 25 (concluded)

	SI	cgsm	cgse	cgs
Volume density of energy of magnetic field	$u=\frac{BH}{2}=\frac{B^2}{2\mu_0\mu_r}=\frac{\mu_0\mu_r H^2}{2}$	$u=\frac{BH}{8\pi}=\frac{B^2}{8\pi\mu_r}=\frac{\mu_r H^2}{8\pi}$	$u=\frac{BH}{8\pi}=\frac{B^2}{8\pi\mu_0\mu_r}=\frac{\mu_0\mu_r H^2}{8\pi}$	$u=\frac{BH}{8\pi}=\frac{B^2}{8\pi\mu_r}=\frac{\mu_r H^2}{8\pi}$
Poynting vector (density of radiant energy flux)	$\mathbf{S}=\mathbf{E}\times\mathbf{H}$	$\mathbf{S}=\frac{1}{4\pi}\mathbf{E}\times\mathbf{H}$		$\mathbf{S}=\frac{c}{4\pi}\mathbf{E}\times\mathbf{H}$
Velocity of electromagnetic waves	$v=\frac{1}{\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu_r}}$	$v=\frac{1}{\sqrt{\varepsilon_0\varepsilon_r\mu_r}}$	$v=\frac{1}{\sqrt{\varepsilon_r\mu_0\mu_r}}$	$v=\frac{c}{\sqrt{\varepsilon_r\mu_r}}$
Maxwell equations				
1. Faraady's law	$\text{rot}\,\mathbf{H}=-\frac{\partial\mathbf{B}}{\partial t}$			$\text{rot}\,\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$
2. Total current law (Ampere's law)	$\text{rot}\,\mathbf{H}=\mathbf{J}+\frac{d\mathbf{D}}{dt}$	$\text{rot}\,\mathbf{H}=4\pi\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}$		$\text{rot}\,\mathbf{H}=\frac{1}{c}\times\left(4\pi\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}\right)$
3. Poisson's equation (Gauss theorem)	$\text{div}\,\mathbf{D}=\rho$	$\text{div}\,\mathbf{D}=4\pi\rho$		
4. Continuity of lines of force of magnetic induction (Gauss theorem)	$\text{div}\,\mathbf{B}=0$			

* All quantities should be measured in units of the relevant system. In particular, the following values should be used for ε_0 and μ_0: in the SI system $\varepsilon_0=8.85\times10^{-12}$ F/m, $\mu_0=1.26\times10^{-6}$ H/m; in the cgsm system $\varepsilon_0=1.11\times10^{-21}$, $\mu_0=1$; in the cgse system $\varepsilon_0=1$, $\mu_0=1.11\times10^{-21}$.

TABLE 26. **Conversion Table—Charge**

	C	cgs (cgse)	cgsm
1 C =	1	3×10^{9}	0.1
1 cgs (cgse) =	3.34×10^{-10}	1	3.34×10^{-11}
1 cgsm =	10	3×10^{10}	1

TABLE 27. **Conversion Table—Field Intensity**

	V/m	V/cm	cgs (cgse)	cgsm
1 V/m =	1	10^{-2}	3.34×10^{-5}	10^{6}
1 V/cm =	10^{2}	1	3.34×10^{-3}	10^{8}
1 cgs (cgse) =	3×10^{4}	3×10^{2}	1	3×10^{10}
1 cgsm =	10^{-6}	10^{-8}	3.34×10^{-11}	1

TABLE 28. **Conversion Table—Surface Density of Charge**

	C/m²	cgs (cgse)	cgsm
1 C/m² =	1	3×10^{5}	10^{-5}
1 cgs (cgse) =	3.34×10^{-6}	1	3.34×10^{-11}
1 cgsm =	10^{5}	3×10^{10}	1

TABLE 29. **Conversion Table—Volume Density of Charge**

	C/m³	cgs (cgse)	cgsm
1 C/m³ =	1	3×10^{3}	10^{-7}
cgs (cgse) =	3.34×10^{-4}	1	3.34×10^{-11}
cgsm =	10^{7}	3×10^{10}	1

TABLE 30. **Conversion Table—Displacement**

	C/m^2	cgs (cgse)	cgsm
1 C/m^2 =	1	3.77×10^{6}	1.26×10^{-4}
1 cgs (cgse) =	2.65×10^{-7}	1	3.34×10^{-11}
1 cgsm =	7.96×10^{3}	3×10^{10}	1

TABLE 31. **Conversion Table—Displacement Flux**

	C	cgs (cgse)	cgsm
1 C =	1	3.77×10^{10}	1.26
1 cgs (cgse) =	2.65×10^{-11}	1	3.34×10^{-11}
1 cgsm =	0.796	3×10^{10}	1

TABLE 32. **Conversion Table—Potential (Potential Difference)**

	V	cgs (cgse)	cgsm
1 V =	1	3.34×10^{-3}	10^{8}
1 cgs (cgse) =	300	1	3×10^{10}
1 cgsm =	10^{-8}	3.34×10^{-11}	1

TABLE 33. **Conversion Table—Capacitance**

	F	cm (cgs, cgse)	cgsm
1 F =	1	8.99×10^{11}	10^{-9}
1 cm (cgs, cgse) =	1.11×10^{-12}	1	1.11×10^{-21}
1 cgsm =	10^{9}	8.99×10^{20}	1

TABLE 34. **Conversion Table—Current Intensity**

	A	cgs (cgse)	cgsm
1 A =	1	3×10^{9}	0.1
1 cgs (cgse) =	3.34×10^{-10}	1	3.34×10^{-11}
1 cgsm =	10	3×10^{10}	1

TABLE 35. **Conversion Table—Resistance**

	Ω	cgs (cgse)	cgsm
1 Ω =	1	1.11×10^{-12}	10^{9}
1 cgs (cgse) =	8.99×10^{11}	1	8.99×10^{20}
1 cgsm =	10^{-9}	1.11×10^{-21}	1

TABLE 36. **Conversion Table—Resistivity**

	Ω·m	Ω·cm	$\frac{\Omega\cdot mm^2}{m}$	cgs (cgse)	cgsm
1 Ω·m =	1	10^{2}	10^{6}	1.11×10^{-10}	10^{11}
1 Ω·cm =	10^{-2}	1	10^{4}	1.11×10^{-12}	10^{9}
1 $\frac{\Omega\cdot mm^2}{m}$ =	10^{-6}	10^{-4}	1	1.11×10^{-16}	10^{5}
1 cgs (cgse) =	8.99×10^{9}	8.99×10^{11}	8.99×10^{15}	1	8.99×10^{20}
1 cgsm =	10^{-11}	10^{-9}	10^{-5}	1.11×10^{-21}	1

TABLE 37. **Conversion Table—Magnetic Induction**

	T	Gs	cgse
1 T =	1	10^{4}	3.34×10^{-7}
1 Gs =	10^{-4}	1	3.34×10^{-11}
1 cgse =	3×10^{6}	3×10^{10}	1

TABLE 38. **Conversion Table—Magnetic Flux**

	Wb	Mx	cgse
1 Wb =	1	10^8	3.34×10^{-3}
1 Mx =	10^8	1	3.34×10^{-11}
1 cgse =	3×10^2	3×10^{10}	1

TABLE 39. **Conversion Table—Magnetic Field Intensity**

	A/m	Oe	cgse	At/cm
1 A/m =	1	1.26×10^{-2}	3.77×10^8	10^{-2}
1 Oe =	79.6	1	3×10^{10}	0.796
1 cgse =	2.65×10^{-9}	3.34×10^{-11}	1	2.65×10^{-11}
1 At/cm = (ampere-turn per centimetre)	10^2	1.26	3.77×10^{10}	1

TABLE 40. **Conversion Table—Magnetomotive Force**

	A	Gb	cgse
1 A =	1	1.26	3.77×10^{10}
1 Gb =	0.796	1	3×10^{10}
1 cgse =	2.65×10^{-11}	3.34×10^{-11}	1

TABLE 41. **Conversion Table—Inductance and Mutual Inductance**

	H	cm (cgs, cgsm)	cgse
1 H =	1	10^9	1.11×10^{-12}
1 cm (cgs, cgsm) =	10^{-9}	1	1.11×10^{-21}
1 cgse =	8.99×10^{11}	8.99×10^{20}	1

TABLE 42. **Conversion Table—Luminance***

	nt	Sb	asb	L
1 nit =	1	10^{-4}	3.14	3.14×10^{-4}
1 stilb =	10^4	1	3.14×10^4	3.14
1 apostilb =	0.318	3.18×10^{-5}	1	10^{-4}
1 lambert =	3.18×10^3	0.318	10^4	1

* The values of the stilb, apostilb and lambert are frequently given in tables not on the basis of the SI candela, but on the basis of the international candle, which is 1.005 times greater (see Sec. 8.3). Here the factors converting the nit into Sb, asb and L should be divided by 1.005 (9.95×10^{-5} Sb, 3.13 asb and 3.13×10^{-4} L), and the factors converting the stilb, apostilb and lambert into nt multiplied by 1.005 (1.005×10^4 nt, 0.320 nt and 3.20×10^3 nt).

TABLE 43. **Values of Relative and Absolute Luminous Efficiency at Different Wavelengths**

γ (Å)	V_γ	η_γ (lm/W)	γ (Å)	V_γ	η_γ (lm/W)
3 800	0.00004	0.03	5 800	0.870	594
4 000	0.0004	0.27	6 000	0.631	431
4 200	0.004	0.73	6 200	0.381	260
4 400	0.023	15.7	6 400	0.175	120
4 600	0.060	41.0	6 600	0.061	41.7
4 800	0.139	90.2	6 800	0.017	11.6
5 000	0.323	221	7 000	0.0041	2.8
5 200	0.710	485	7 200	0.00105	0.72
5 400	0.954	652	7 400	0.00025	0.17
5 600	0.995	680	7 600	0.00006	0.04

TABLE 44. **Relationship between Electron-Volt and Other Units**

Defining relationship	Unit	1 eV =	Reciprocal expressing given unit in eV
$E=eU$	J	1.60×10^{-19}	6.24×10^{18}
	erg	1.60×10^{-12}	6.24×10^{11}
$\frac{3}{2}kT=eU$	°K	7.73×10^3	1.29×10^{-4}
$kT=eU$	°K	1.16×10^4	8.62×10^{-5}
$Q=eUN_A$	cal/mole (kcal/kmole)	2.31×10^4	4.36×10^{-5}
$\frac{hc}{\gamma}=eU$	cm^{-1}	8.07×10^3	1.24×10^{-4}
$h\nu=eU$	s^{-1}	2.42×10^{14}	4.13×10^{-15}
1 amu $c^2=eU$	e	1.07×10^{-9}	9.32×10^8
$m_ec^2=eU$	m_e	1.95×10^{-6}	5.11×10^5
$Rch=eU$	R_y	7.35×10^{-2}	13.6

TABLE 45. **Conversion Table—Effective Cross Section**

		m^2	cm^2	barn	a_0^2	πa_0^2	$cm^2/cm^3 \times$ $\times$ mm Hg
1 m^2	=	1	10^4	10^{28}	3.57×10^{20}	1.15×10^{20}	3.54×10^{20}
1 cm^2	=	10^{-4}	1	10^{24}	3.57×10^{16}	1.15×10^{16}	3.54×10^{16}
1 barn	=	10^{-28}	10^{-24}	1	3.57×10^{-8}	1.15×10^{-8}	3.54×10^{-8}
a_0^2	=	2.80×10^{-21}	2.80×10^{-17}	2.80×10^{7}	1	0.318	0.991
πa_0^2	=	8.80×10^{-21}	8.80×10^{-17}	8.80×10^{7}	3.14	1	3.11
$1\,\frac{cm^2}{cm^3}$ mm Hg	=	2.83×10^{-21}	2.83×10^{-17}	2.83×10^{7}	1.01	0.321	1

TABLE 46. **Beaufort Scale**

Beaufort number	Velocity, m/s	Beaufort number	Velocity, m/s
0	0-0.5	7	12.5-15.2
1	0.6-1.7	8	15.3-18.2
2	1.8-3.3	9	18.3-21.5
3	3.4-5.2	10	21.6-25.1
4	5.3-7.4	11	25.2-29.0
5	7.5-9.8	12	>29.0
6	9.9-12.4		

TABLE 47. **Hardness Scales**

Mineral	Hardness number		Mineral	Hardness number	
	Mohs' scale	Breit-haupt's scale		Mohs' scale	Breit-haupt's scale
Talc	1	1	Hornblende	—	7
Gypsum	2	2	Felspar	6	8
Mica	—	3	Quartz	7	9
Lime felspar	3	4	Topaz	8	10
Fluorite	4	5	Corundum	9	11
Apatite	5	6	Diamond	10	12

TABLE 48. **Some Most Frequently Encountered British and U.S. Units**

Acre—a unit of area, equal to 4046.86 m^2.

Barrel—a unit of volume (capacity). There are distinguished a dry barrel, equal to 115.628 l, and a petroleum oil barrel, equal to 158.988 l.

B.t.u. (British thermal unit)—a unit of work or energy, equal to 1.055×10^3 J.

Bushel—a unit of volume (capacity). A U.K. bushel equals 36.3687l, a U.S. bushel equals 35.2393 l.

Gallon—a unit of volume (capacity). A U.K. gallon equals 4.546091, a U.S. gallon equals 3.78543 l.

Table 48 (concluded)

Grain — a unit of weight, equal to 0.0648 g.
Mil — a thousandth of an inch, equal to 2.54 μ.
Mile (U.K.) — 1609.344 m.
Ounce (Avoirdupois) — a unit of mass (1/16 U.K. pound), equal to 28.3495 g.
Pound — a unit of mass, equal to 453.5924 g.
Poundal — a unit of force, equal to $1{,}38255 \times 10^4$ dyn.
Quart — a unit of volume (capacity). A U.K. quart equals 1.13650 l, a U.S. dry quart equals 1.1012 l, and a liquid quart equals 0.94633 l.
Ton — a unit of mass. A U.K. (long) ton equals 2240 pounds or 1016 kg; a U.S. (short) ton equals 2000 pounds or 907.2 kg.
Yard — a unit of length, equal to 3 feet or 0.9144 m.

TABLE 49. **Some Units and Names of Units Having a Limited Use or Not Introduced Officially**

Acohm — acoustic ohm — cgs unit of acoustic resistance.
Biot — cgsm unit of current intensity, equal to 10 A.
Eötvös — unit of the gradient of the force of gravity, equal to a change in the acceleration of gravity by 1 cm/s^2 per centimetre.
Franklin — cgs unit of charge, equal to 3.34×10^{10} C.
Gal — unit of acceleration, equal to 1 cm/s^2.
Inerta — unit of mass in the mk(force)s system, equal to 9.81 kg.
Kayser — unit of wave number, equal to cm^{-1}.
Kilopond — name of kilogram-force used in German literature.
Lenz — unit of magnetic field intensity, equal to 1 A/m.
Magn — unit of "absolute" permeability, equal to $10^7/4\pi$ F/m.
Mes — unit of velocity, equal to 1 m/s.
Mho — unit of conductivity. The same as siemens.
Pascal — unit of pressure equal to 1 N/m^2.
Point (of the compass) — a unit of angle used in navigation, equal to 1/32 of a circle, i. e., 11.25°.
Rhe — cgs unit of fluidity. The fluidity of a liquid having a viscosity of 1 poise.
Uranium unit — a unit of α-activity — the activity of the oxide U_3O_8 with a density of surface coating of 20 mg/cm^2. A uranium unit creates an ionization current in air having a density of 5.78×10^{-13} A/cm^2.

TABLE 50. **Temperature Points**

A number of temperature points supplementing the reference points given in Sec. 5.3 have been selected for practical reproduction of separate sections of the thermodynamic temperature scale. These supplementary temperature points, the same as the reference ones, are the points of equilibrium of two or three phases of a given substance. A number of such points are given below, it being indicated between what phases equilibrium serves for establishing the given point. In the equilibrium of two phases the pressure is equal to a standard atmosphere. The temperatures are given in °C.

Carbon dioxide—solid and vapour	−78.5	Mercury—liquid and vapour	356.58
Mercury—solid and liquid	−38.87	Aluminium—solid and liquid	650.1
Ice and water	0.000	Copper—solid and liquid	1 083
Diphenyl oxide—triple point	26.88	Nickel—solid and liquid	1 453
Benzoic acid—triple point	122.36	Cobalt—solid and liquid	1 492
Indium—solid and liquid	156.61	Palladium—solid and liquid	1 552
Naphthalene — liquid and vapour	218.0	Platinum—solid and liquid	1 768
Tin—solid and liquid	231.91	Rhodium—solid and liquid	1 960
Benzophenone—liquid and vapour	305.9	Iridium—solid and liquid	2 443
Cadmium—solid and liquid	321.03		
Lead—solid and liquid	327.3		

TABLE 51. **Symbols of Units**

Unit	Symbol	Unit	Symbol	Unit	Symbol
Ampere	A	Bar	bar	Coulomb	C
Angstrom	Å	Barn	b	Curie	c
Apostilb	asb	Bel	B	Day	d
Are	a	Bit	bit	Debye	D
Atmosphere (standard)	atm	Bohr magneton	μ_B	Decibel	dB
Atmosphere (technical)	at	Calorie	cal	Degree	°K (deg)
Atomic mass unit	amu	Candela	cd	Dioptre	D
		Cent	c	Dyne	dyn
		Centimetre	cm	Einstein	E
				Electron-volt	eV

Table 51 (concluded)

Unit	Symbol	Unit	Symbol	Unit	Symbol
Erg	erg	Mile (nautical)	n. mile	Rad	rad
Farad	F	Millimetre of mercury	mm Hg	Radian	rad
Fermi	f	Millimetre of water	mm H_2O	Roentgen	r
Gal	G	Minute	min	Rutherford	Rd
Gamma	γ	Minute (angular)	′	Rydberg	R_y
Gauss	Gs	Mole	mole	Savart	Sav
Gilbert	Gb	Neper	n	Second	s
Gram	g	Newton	N	Second, angular	″
Henry	H	Nit	nt	Siemens	S
Hertz	Hz	Octave	oct	Steradian	sr
Horsepower	hp	Oersted	Oe	Sthene	sn
Hour	h	Ohm	Ω	Stilb	Sb
Joule	J	Parsec	pc	Stokes	St
Kilogram	kg	Phon	P	Tesla	T
Knot	kn	Phot	ph	Ton	ton
Lambert	L	Pieze	pz	Torr	torr
Litre	l	Poise	P	Volt	V
Lumen	lm	Quintal	q	Watt	W
Lux	lx			Weber	wb
Maxwell	Mx			X unit	XU
Metre	m				
Micron (micrometre)	μ (μm)				

TABLE 52. **Prefixes for Multiples and Submultiples of Units**

Name	Multiple of basic unit	Symbol	Example	
Tera	10^{12}	T	terajoule	TJ
Giga	10^{9}	G	giganewton	GN
Mega	10^{6}	M	megaohm (megohm)	M
Kilo	10^{3}	k	kilogauss	kGs
Hecto	10^{2}	h	hectowatt	hW
Deca	10	da	decalitre	dal
Deci	0.1	d	decimetre	dm
Centi	10^{-2}	c	centipoise	cP
Milli	10^{-3}	m	milliampere	mA
Micro	10^{-6}	μ	microvolt	μV
Nano	10^{-9}	n	nanosecond	ns
Pico	10^{-12}	p	picofarad	pF
Femto	10^{-15}	f	femtogram	fg
Atto	10^{-18}	a	attocoulomb	aC

TABLE 53. **Symbols of Physical Quantities**

The present table includes quantities encountered in the general course of physics and related subjects. The symbols used are generally those recommended by international organizations for various branches of science and engineering, or have been taken from the most well known textbooks on physics.

According to existing rules, capital and lower case letters may be interchanged where this is expedient and does not lead to confusion.

Quantity	Symbol	Quantity	Symbol
Acceleration	a	Dielectric constant (relative permittivity)	ε_r
Acceleration, angular	α	Difference of potentials	U
Action	S, H	Displacement	D
Activity of radioactive source	A	Efficiency	η
Amplitude	A	Electric constant (permittivity of vacuum)	ε_0
Angle	α, φ, ψ, θ	Energy	E, W
Area	A	Energy, free	F
Avogadro's number	N_A	Energy, internal	U
Boltzmann's constant	k	Energy, kinetic	E_k
Capacitance	C	Energy, potential	E_p
Charge, electric	Q	Entropy	S
Charge of electron	e	Faraday's constant	F
Coefficient, friction	C_{fr}, f	Field intensity, electrical	E
Coefficient, heat transfer	α	Field intensity, magnetic	H
Coefficient, recombination	A	Fluidity	φ
Concentration	n	Flux, electric	N_D
Conductance	G	Flux, luminous	Φ
Curvature	ρ	Flux, magnetic	Φ
Curvature, Gaussian	K	Flux, radiant	Φ_r
Density	ρ	Flux linkage	Ψ
Density, charge, linear	τ	Force	F, P, Q, R
Density, charge, surface	σ	Force, electromotive	$\mathscr{E}$
Density, charge, volume	ρ	Force, magnetomotive	F
Density, electric current	J	Frequency	ν, f
Density, energy	e	Frequency, angular	ω
Density, energy, volume	u	Gas constant	R
Diameter	D, d	Gravitational constant	G

Table 53 (continued)

Quantity	Symbol	Quantity	Symbol
Heat flow	Φ	Moment of inertia (polar)	I_0
Heat of phase conversion (fusion, evaporation)	H, r	Moment, magnetic	$\vec{p}_m$
Illumination	E	Moment of momentum (angular momentum)	$\mathscr{L}$
Impedance	Z	Moment, statical	S
Impulse	p	Momentum	$\bar{p}$ (mv)
Inductance	L	Musical interval	I_m
Inductance, mutual	M	Number	n, N
Induction, magnetic	B	Period	T, τ
Intensity, current	I	Permeability, relative	μ_r
Intensity, luminous	I	Permittivity, relative	ε_r
Intensity, sound	I	pH index	pH
Length	l	Planck constant	h
Length of free path	l	Planck-Dirac constant	$\hbar$
Length of light wave	λ	Polarization, dielectric	$\vec{p}$
Level, sound loudness	L_N	Polarization, of molecule	α
Level, sound pressure	L_p	Potential	V
Luminance	L	Power	P
Luminous efficiency, absolute	η	Power, lens	D
Luminous efficiency, relative	V	Poynting vector	$\mathbf{S}$
Luminous emittance	R	Pressure	p
Magnetic constant (permeability of vacuum)	μ_0	Quantity of heat	Q
Magnetization	J	Quantity of illumination	H
Mass	m	Quantity of light	Q
Mass, electron	m_e	Radiation dose	D
Mass, relative molecular (molecular weight)	M	Radius	r
Mass flow	Q_m	Reflection factor	ρ
Mobility	b	Refraction index	n
Modulus, elasticity (Young's modulus)	E	Reluctance	R_m
Modulus, shear	G	Resistance, acoustic	R_a
Moment, dipole	$\vec{p}$	Resistance, electrical	R
Moment of force	M	Resistance, mechanical, of acoustic system	R_m
Moment of inertia (axial)	J_z	Resistivity	ρ
Moment of inertia (dynamic)	I	Solid angle	Ω

Table 53 (concluded)

Quantity	Symbol	Quantity	Symbol
Specific heat	c	Velocity (speed)	v
Spin number	s	Velocity, angular	ω
Surface tension	σ	Velocity, of light	c
Susceptibility, dielectric	χ_e	Viscosity, dynamic	η
Susceptibility, magnetic	χ_m	Viscosity, kinematic	ν
Temperature	$t°$, θ	Volume	V
Temperature, absolute	T	Volumetric flow rate	Q_V
Thermal diffusitivity	a	Wave number	$\bar{\nu}$, σ
Time	t, τ	Weight	G
		Weight, specific	γ
		Work	W

TABLE 54. **Modulus of Elasticity (Young's Modulus) for Selected Materials (Mean Rounded off Values)**

Material	E	Material	E
Aluminium	7	Rubber	0.5
Copper	12	Quartz	5
Steel	20	Lead	1.6

To obtain the values of the modulus in N/m^2 the numbers in the E column should be multiplied by 10^{10}, in dyn/cm^2—by 10^{11}, in kgf/m^2—by 10^9, in kgf/cm^2—by 10^5 and in kgf/mm^2—by 10^3.

TABLE 55. **Viscosity of Selected Liquids at 20°C**

Liquid	Viscosity in $\frac{N \cdot s}{m^2} \times 10^3$	Liquid	Viscosity in $\frac{N \cdot s}{m^2} \times 10^3$
Ether	0.23	Ethyl alcohol	1.19
Methyl alcohol	0.59	Mercury	1.59
Benzene	0.65	Glycerine	850
Water	1.01		

TABLE 56. **Specific Heats of Selected Substances**

Substance	Specific heat, $J/(kg \cdot deg) \times 10^{-2}$	Specific heat, $cal/(g \cdot deg)$	Substance	Specific heat, $J/(kg \cdot deg) \times 10^{-2}$	Specific heat, $cal/(g \cdot deg)$
Aluminium	8.8	0.21	Water	41.9	1.00
Iron	4.6	0.11	Quartz	8.4	0.20
Copper	3.8	0.091	Glass	6.3	0.15
Germanium	3.1	0.074	Mercury	1.3	0.033
Tungsten	1.5	0.036			

TABLE 57. **Thermal Conductivities of Selected Materials**

Material	Thermal conductivity, $\frac{W}{m \cdot deg}$	Thermal conductivity, $\frac{cal}{s \cdot cm \cdot deg}$	Thermal conductivity, $\frac{kcal}{h \cdot m \cdot deg}$	Material	Thermal conductivity, $\frac{W}{m \cdot deg}$	Thermal conductivity, $\frac{cal}{s \cdot cm \cdot deg}$	Thermal conductivity, $\frac{kcal}{h \cdot m \cdot deg}$
Copper	390	0.92	330	Cement	2.9	7×10^{-3}	2.5
Aluminium	210	0.51	190	Brick	1.7	4.1×10^{-3}	1.4
Graphite	130	0.30	110	Glass	0.84	2×10^{-3}	0.7
Brass	110	0.26	94	Water	0.63	1.5×10^{-3}	0.54
Tungsten	76	0.18	65	Cotton	0.25	6×10^{-4}	0.22
Steel	46	0.11	40	Wood	0.21	5×10^{-4}	0.18
Mercury	6.7	1.6×10^{-2}	5.8	Felt	0.038	9×10^{-5}	0.032

TABLE 58. **Acoustic Resistivity of Selected Media**

Medium	Acoustic resistivity in $g/s \cdot cm^2$	Medium	Acoustic resistivity in $g/s \cdot cm^2$
Air (in standard conditions)	43	Steel	4.1×10^6
Mercury	2.0×10^6	Copper	3.2×10^6
Water	1.4×10^5	Rubber	2.9×10^3

TABLE 59. Resistivity of Selected Conductors

Conductor	Resistivity $\Omega \times \frac{\text{area}}{\text{length}}$	Resistivity cgs $\times 10^{18}$	Conductor	Resistivity $\Omega \times \frac{\text{area}}{\text{length}}$	Resistivity cgs $\times 10^{18}$
Bismuth	120	130	Molybdenum	4.8	5.3
Nichrome	110	120	Aluminium	3.2	3.6
Manganine	43	48	Lead	2.1	2.3
Steel	20	22	Copper	1.8	2.0
Tantalum	15	17	Silver	1.6	1.8
Brass	8	8.9			

To obtain the values of the resistivity in $\Omega \cdot m$, $\Omega \cdot cm$ and $\Omega \cdot \frac{mm^2}{m}$, the numbers in the column $\Omega \times \frac{\text{area}}{\text{length}}$ should be multiplied respectively by 10^{-8}, 10^{-6} and 10^{-2}.

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